

MATH 550B, HOMEWORK 6

VAN KAMPEN'S THEOREM

Due by midnight, Tuesday, 11/4

Reading. Read Hatcher pages 40 to 50.

Problems (to turn in).

- (1) A) Show that if A is a deformation retract of X then A is homotopic to X .
B) Let $X = S^1$ and $Y = \{(x, y) | x^2 + y^2 = 1\} \cup \{(x, y) | y = 0, 1 < x < 2\}$. Show X is homotopic to Y .
- (2) Use van Kampen's theorem and induction to show $\pi_1(S^n, x_0) \cong \{1\}$ for $n \geq 2$. Hint: S^n can be constructed by gluing two n -balls together along their boundary.
- (3) Hatcher page 53 exercise 8.
- (4) Hatcher page 53 exercise 10. Hint: find an expression for γ in terms of the generators of $\pi_1(D^2 \times I \setminus \{\alpha \cup \beta\})$.