Math 550A, Homework 7

Quotient Spaces

Not to be turned in

Exercises (to do on your own)

1. Prove that the quotient topology actually defines a topology.
2. Define an equivalence relation on $X = \mathbb{R}^2$ by setting
$$x_1 \times y_1 \sim x_2 \times y_2 \quad \text{if} \quad y_1 - (x_1)^3 = y_2 - (x_2)^3.$$
Let X^* be the quotient space $X/\sim$. To what familiar space is X^* homeomorphic? (Give a convincing argument.)
3. Answer the same question as above for the equivalence relation
$$x_1 \times y_1 \sim x_2 \times y_2 \quad \text{if} \quad (x_1)^2 + (y_1)^2 = (x_2)^2 + (y_2)^2.$$
4. Define an equivalence relation on $X = \mathbb{R}^n - \{\mathbf{0}\}$ by setting
$$\mathbf{x} \sim \mathbf{y} \quad \text{if} \quad \mathbf{x} = \lambda \mathbf{y} \quad \text{where} \quad \lambda > 0.$$
Let X^* be the quotient space $X/\sim$. To what familiar space is X^* homeomorphic?
5. Let X be the topological space $\mathbb{R}^2$ with the dictionary order topology. Define an equivalence relation on X by setting
$$(x_1, y_1) \sim (x_2, y_2) \quad \text{if} \quad x_1 = x_2.$$
Let X^* be the quotient space $X/\sim$. To what familiar space is X^* homeomorphic?
6. Answer the same question as above for the equivalence relation
$$(x_1, y_1) \sim (x_2, y_2) \quad \text{if} \quad y_1 = y_2.$$
Let X^* be the quotient space $X/\sim$. To what familiar space is X^* homeomorphic?
7. Let X be the topological space $\mathbb{R}^2$ with the product topology. Define an equivalence relation on X by setting
$$(x_1, y_1) \sim (x_2, y_2) \quad \text{if} \quad x_1 = x_2 mod 1 \quad \text{and} \quad y_1 = y_2 mod 1.$$
Let X^* be the quotient space $X/\sim$. To what familiar space is X^* homeomorphic?