Math 500, Homework 2

Closed sets, T_1 and Hausdorff spaces, continuous functions

Due at start of class, Tuesday, 10/4

Reading Read §17 – 18 of Munkres.

Exercises (to do on your own)

1. Munkres §17, exercise 16 (only for $\mathbb{R}$, $\mathbb{R}_\ell$ and $\mathbb{R}_K$).
2. Prove: a product of two Hausdorff spaces is Hausdorff.
3. Prove: a subspace of a Hausdorff space is Hausdorff.
4. Show that the subspace $(a, b) \subset \mathbb{R}$ with $a < b$ is homeomorphic to $(0, 1)$.
5. Define S^1 to be the following subset of $\mathbb{R}^2$:

$$S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}.$$

   Prove that S^1 is closed by using the fact that the function $f : \mathbb{R}^2 \to \mathbb{R}$, where $f(x, y) = x^2 + y^2$, is continuous.

6. (Infinite pasting lemma?) Suppose $\{A_\alpha\}$ is a collection of closed subsets of X whose union is X and $f : X \to Y$ is a map such that every restriction map $f|_{A_\alpha} : A_\alpha \to Y$ is continuous. Must f be continuous?

Problems (to turn in)

1. Munkres §17, exercise 6. (I also suggest doing §17, exercise 7, but you don't need to write it up.)
2. A topological space X is T_1 (as we defined this in the lecture) if and only if all one-point sets $\{x\}$, where $x \in X$, are closed.
3. Munkres §18, exericse 1.
4. Munkres §18, exercise 8 (replace Y with $\mathbb{R}$ if you would like).
5. Munkres §18, exercise 11. Be sure to also read exercise 12.