MATH 233, HOMEWORK 8

SETS WITH STRUCTURE AND PARTITIONS

Due by 10 am, Friday, April 12th

1. Homework Policy

You are strongly encouraged to work in groups to exchange ideas and help each other understand how to approach problems, but the work you turn in must be your own! If you work with others on an assignment, be sure to indicate the names of the other students on your homework. Additionally, if you use any outside resources (i.e. internet sources, other mathematicians, other books) to help you solve homework problems, you must cite your sources. Failure to follow these rules will result in a score of zero on an assignment and may constitute a violation of academic integrity.

Homework must be legible, well-organized, and written in complete sentences. Handwritten work is fine, but you are encouraged to type up the problems in LaTeX.

Additional guidelines: If you submit hand written work make sure it is written legibly and stapled. If you submit work through email mail, it must me submitted as a **single pdf file** and have your name on the first page. Failure to follow these guidelines with result in a loss of points.

2. Readings and Responses.

(1) Read Sections 5.6, 5.7 and 7.1.
(2) Write a response to exercise 5.6.7. You do not need to give a careful proof, only a quick written argument.
(3) Let $S = \{(x, y) \in \mathbb{R}^2 : y = x^2 \text{ and } y \leq 4\}$. Sketch a picture of $S \times [0, 1]$.
(4) Let $W = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1 \text{ and } z \geq 0\}$. Sketch a picture of $W \times [0, 1]$.

3. Problems

(1) Let $\{0, 1, 2, 3\}$ be a group with identity 0 and operation $\circ$ defined by $0 \circ 0 = 0$, $0 \circ 1 = 1$, $0 \circ 2 = 2$, $0 \circ 3 = 3$, $1 \circ 0 = 1$, $1 \circ 1 = 0$, $1 \circ 2 = 3$, $1 \circ 3 = 2$, $2 \circ 0 = 2$, $2 \circ 1 = 3$, $2 \circ 2 = 0$, $2 \circ 3 = 1$, $3 \circ 0 = 3$, $3 \circ 1 = 2$, $3 \circ 2 = 1$, $3 \circ 3 = 0$. Show that $\{0, 1\}$ is a subgroup of this group.
(2) Find a counter example to the following statement. Prove that the example you find is a counter example: Suppose G is a group with operation $\circ$ and identity e. If H and K are both subgroups of G, then $H \cup K$ is a subgroup of G. (Hint: Use the previous exercise to help you build a counterexample)
(3) Do Exercise 7.1.4 part 1.
(4) Do Exercise 7.1.5.