

MATH 123 PRACTICE FINAL EXAM SPRING 2016

1. (10 pts) Evaluate the following integral

$$\int \tan^{-1}(x) dx$$

2. (10 pts) Show that the following improper integral converges.

$$\int_0^{\infty} \frac{\tan^{-1}(x)}{e^x} dx$$

3. (10 pts) The Nights Watch is repairing the wall after a skirmish with wildling raiders. They must use a chain to lift a 2000lb block of solid ice vertically from base of the wall to the top of the wall. The wall is 700ft high and the team of builders takes 8 hours to hull the ice block to the top. The chain they use to hull the ice up weighs a pound a foot and the ice block melts at a rate of 50lb per hour. How much work is required to lift the chain and ice block to the top of the wall?

4. (10 pts) Find the interval of convergence for the following power series.

$$\sum_{n=1}^{\infty} \frac{(4x - 2)^n \ln(n)}{n}$$

5. (10pts) Show that the following series converges conditionally.

$$\sum_{n=3}^{\infty} \frac{(-1)^n}{n \cdot \ln(n)}$$

6. (10 pts) Use the definition of Taylor series to find the Taylor series for $y = xe^x$ at $x = 0$.

7. (10 pts)

A) Find a representation of the polar curve $r = f(\theta)$ as a parametric curve.

B) Use the representation from part A) to find a formula for the slope of the curve $r = f(\theta)$.

C) Use the formula that you found in part B) to find the slope of the curve $r = \theta$ when $\theta = \frac{\pi}{2}$.

8. Let $R_n(x)$ be the error in approximating $f(x) = xe^x$ by the n-th Taylor polynomial centered at $x = 0$. Find the smallest value of n such that $|R_n(x)| < \frac{1}{1000}$ for all x in the interval $[-1, 1]$.