Math 123: Polar Coordinates

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Outline

1 Polar Coordinates

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Example: Derive these conversion rules.
Example: Sketch the graph of $r = cos(2\theta)$.

Derivatives of Parametric Curves

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So, if $r = f(\theta)$ is a curve in polar coordinates, then

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Example: Find the slope of the curve $r = cos(2\theta)$ at $\theta = \frac{\pi}{4}$.
Example: Find the points on the curve $r = e^{\theta}$ where the tangent line is horizontal or vertical.

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Example: Find the area enclosed by one leaf of $r = cos(2\theta)$.