Math 123: Integration By Parts and Trig Integrals

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Outline

Integration by Parts

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

Exercise: Derive the above equality by using the product rule to find the derivative of $u(x)v(x)$.

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Example: Find $\int xe^x dx$.
Key: Let u be the function that gets simpler as you differentiate.
Example: Find $\int x^2 sin(x)dx$.
Key: Multiple iterations may be necessary.
Example: Find $\int cos(x)e^x dx$.

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Example: Find $\int ln(x)dx$.

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Key: Sometimes by-parts can be used to integrate functions we know how to differentiate.

How do we integrate Trigonometric functions?

Example: $\int sin(x)dx$ and $\int cos(x)dx$.

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$$cos^2(x) + sin^2(x) = 1$$

and u-substitution

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Example: $\int cos^5(x)dx$

How do we integrate Trigonometric functions?

Example: $\int \sin^2(x)\,dx$

How do we integrate Trigonometric functions?

Example: $\int \sin^2(x)dx$

For $\int \cos^{even}(x)dx$ or $\int \sin^{even}(x)dx$ use

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

possibly multiple times

How do we integrate Trigonometric functions?

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Example: $\int cos^4(x)dx$

How do we integrate Trigonometric functions?

Example: $\int \sin^2(x) \cos^3(x) dx$.

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Example: $\int \sin^2(x) \cos^3(x) dx$.

For $\int \sin^{anything}(x) \cos^{odd}(x) dx$ or $\int \cos^{anything}(x) \sin^{odd}(x) dx$ use

$$\cos^2(x) + \sin^2(x) = 1$$

and u-substitution

How do we integrate Trigonometric functions?

Example: $\int \sin^2(x)\cos^3(x)dx$.

For $\int \sin^{anything}(x)\cos^{odd}(x)dx$ or $\int \cos^{anything}(x)\sin^{odd}(x)dx$ use

$$\cos^2(x) + \sin^2(x) = 1$$

and u-substitution
Question: What about $\int \sin^{even}(x)\cos^{even}(x)dx$

How do we integrate Trigonometric functions?

Example: $\int tan(x)sec^4(x)dx$.

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Example: $\int \tan(x)\sec^4(x)dx$.

For integrals involving $\tan(x)$ and $\sec(x)$ use

$$1 + \tan^2(x) = \sec^2(x)$$

and u-substitution.

Some Challenges

Example: Find $\int sec(x)dx$.
Example: Find $\int sec^3(x)dx$.