Math 123: Volumes and Arc Length

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Outline

Volumes of solids of rotation

Replace all x's with y's in the following formulas to get other valid expressions for volume.

Disks:

Vol $= \int_a^b \pi(\text{radius in terms of x})^2 dx$

Shells:

Vol $= \int_a^b 2\pi(\text{radius in terms of x})(\text{height in terms of x}) dx$

Washers:

Vol $=$
$\int_a^b \pi(\text{outer radius in terms of x})^2 - \pi(\text{inner radius in terms of x})^2 dx$

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Exercise: Find the volume of the object obtained by rotating the region bounded by the lines $y = x$, $y = 1$ and $x = 0$ about the x-axis.

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Exercise: Find the volume of the object obtained by rotating the region bounded by the lines $y = x$, $y = 1$ and $x = 0$ about $x = -2$.

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Exercise: Find the volume of the object obtained by rotating the region bounded by the curves $y = cos(x) + 1$ and $y = 0$ that contains $(2\pi, 1)$ about the x-axis.

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Exercise: Find the volume of the object obtained by rotating the region bounded by the curves $y = cos(x) + 1$ and $y = 0$ that contains $(2\pi, 1)$ about the y-axis.

The length of a curve

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Example: Find circumference of the circle $x^2 + y^2 = 4$.

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Example: Find circumference of the circle $x^2 + y^2 = 4$.
Example: Find the length of the curve $y = ln(cos(x))$ between $x = 0$ and $x = \frac{\pi}{3}$