Math 123: Operations on Power Series

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Outline

Review

Definition

A **Power Series** is a series and a function of the form

$$P(x) = \Sigma_{k=0}^{\infty} c_k (x-a)^k = c_1 + c_2(x-a) + c_3(x-a)^2 + \ldots$$

The radius of convergence is a positive number R such that $P(x)$ converges for x such that $|x-a| < R$.

$$R = lim_{k\to\infty} \left|\frac{c_k}{c_{k+1}}\right|$$

Interval of Convergence

Given a power series $\Sigma_{k=0}^{\infty} c_k (x-a)^k$ with radius of convergence R, the **interval of convergence** is one of the following where we include endpoints if the series is convergent at those points.

$$(a-R, a+R), [a-R, a+R), (a-R, a+R], [a-R, a+R]$$

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Exercise: Find the interval of convergence of $\sum_{k=1}^{\infty} \frac{(x)^k}{k}$.
Exercise: Find the radius of convergence for the power series $\sum_{k=1}^{\infty} \frac{(3x-3)^k}{k5^k}$.

Using the geometric series

Exercise: Use

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

to find a power series for $f(x) = \frac{1}{1+x^2}$ and find the interval of convergence.

Derivatives and Integrals of Series

Theorem

If $P(x) = \Sigma_{k=0}^{\infty} c_k (x-a)^k$, *then*

$$P'(x) = \Sigma_{k=1}^{\infty} k c_k (x-a)^{k-1}$$

$$\int P(x)dx = C + \Sigma_{k=0}^{\infty} \frac{c_k}{k+1} (x-a)^{k+1}$$

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Exercise: Find the derivative of $\Sigma_{n=0}^{\infty} \frac{x^n}{n!}$.

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Exercise: If R is the radius of convergence of $P(x)$ find the radius of convergence of $P'(x)$.

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Exercise: Find the derivative of $\Sigma_{n=0}^{\infty} \frac{x^n}{n!}$.
Exercise: If R is the radius of convergence of $P(x)$ find the radius of convergence of $P'(x)$.
Exercise: Find the power series for $f(x) = ln(1+x)$.