Math 123: Comparison Tests for Series

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Outline

The Comparison Test

Theorem

Let $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ be positive series.

1. *If $a_i \leq b_i$ for all i and $\sum_{i=1}^{\infty} b_i$ converges, then $\sum_{i=1}^{\infty} a_i$ converges.*
2. *If $a_i \leq b_i$ for all i and $\sum_{i=1}^{\infty} a_i$ diverges, then $\sum_{i=1}^{\infty} b_i$ diverges.*

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2. *If $a_i \leq b_i$ for all i and $\Sigma_{i=1}^{\infty} a_i$ diverges, then $\Sigma_{i=1}^{\infty} b_i$ diverges.*

Determine if $\Sigma_{i=2}^{\infty} \frac{\sqrt{i}}{2i-1}$ is convergent or divergent.

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2. *If $a_i \leq b_i$ for all i and $\sum_{i=1}^{\infty} a_i$ diverges, then $\sum_{i=1}^{\infty} b_i$ diverges.*

Determine if $\sum_{i=2}^{\infty} \frac{\sqrt{i}}{2i-1}$ is convergent or divergent.
Determine if $\sum_{i=2}^{\infty} \frac{1}{i!}$ is convergent or divergent.

Limit Comparison Test

Theorem

Let $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ be positive series. If

$$lim_{i\to\infty}\frac{a_i}{b_i} = C$$

where C is a finite positive constant, then either both $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ converge or both $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ diverge.

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Determine if $\Sigma_{i=1}^{\infty} \frac{i+2}{(i+1)^3}$ is convergent or divergent.

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Determine if $\Sigma_{i=1}^{\infty}\frac{i+2}{(i+1)^3}$ is convergent or divergent.

Determine if $\Sigma_{i=1}^{\infty}\frac{2i^2-1}{i^2 3^i}$ is convergent or divergent.