

## Introduction

In the next three chapters, we will examine different aspects of capital market theory, including:

- Bringing risk and return into the picture of investment management – Markowitz optimization
- Modeling risk and return – CAPM, APT, and variations
- Estimating risk and return – the Single-Index Model (SIM) and risk and expected return factor models
- These provide the framework for both modern finance, which we have briefly discussed already, as well as for quantitative investment management, which will be the subject of the next section of the course

## Lecture Presentation Software

to accompany

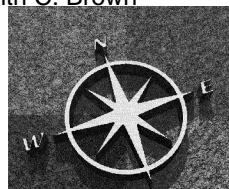
### *Investment Analysis and Portfolio Management*

Seventh Edition

by

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## Chapter 7



## Chapter 7 - An Introduction to Portfolio Management

Questions to be answered:

- What do we mean by *risk aversion* and what evidence indicates that investors are generally risk averse?
- What are the basic assumptions behind the Markowitz portfolio theory?
- What is meant by *risk* and what are some of the alternative measures of risk used in investments?

## Chapter 7 - An Introduction to Portfolio Management

- How do you compute the expected rate of return for an individual risky asset or a portfolio of assets?
- How do you compute the standard deviation of rates of return for an individual risky asset?
- What is meant by the covariance between rates of return and how do you compute covariance?

## Chapter 7 - An Introduction to Portfolio Management

- What is the relationship between covariance and correlation?
- What is the formula for the standard deviation for a portfolio of risky assets and how does it differ from the standard deviation of an individual risky asset?
- Given the formula for the standard deviation of a portfolio, how and why do you diversify a portfolio?

## Chapter 7 - An Introduction to Portfolio Management

- What happens to the standard deviation of a portfolio when you change the correlation between the assets in the portfolio?
- What is the risk-return efficient frontier?
- Is it reasonable for alternative investors to select different portfolios from the portfolios on the efficient frontier?
- What determines which portfolio on the efficient frontier is selected by an individual investor?

## Background Ideas

- As an investor you want to maximize the returns for a given level of risk.
- The relationship between the returns for the different assets in the portfolio is important.
- A good portfolio is not simply a collection of individually good investments.

## Risk Aversion

Given a choice between two assets with equal rates of return, most investors will select the asset with the lower level of risk.

Risk aversion is a consequence of decreasing marginal utility of consumption.

## Evidence That Investors are Risk-Averse

- Many investors purchase insurance for: Life, Automobile, Health, and Disability Income. The purchaser trades known costs for unknown risk of loss.
- Yields on bonds increase with risk classifications, from AAA to AA to A ...
- Lottery tickets seemingly contradict risk aversion, but provide potential for purchasers to move into a new class of consumption.

## Definition of Risk

But, how do you actually define *risk*?

1. Uncertainty of future outcomes
- or
2. Probability of an adverse outcome

## Markowitz Portfolio Theory

- Old adage is:
  - “Don’t put all your eggs in one basket.”
  - But, how many baskets should you use?
  - And how what proportion of your eggs should you put in each basket?
- Harry Markowitz
  - wrestled with these questions
  - figured out a way to answer both of them
  - Earned a Nobel Prize in Economics (1990) for his efforts

## Markowitz Portfolio Theory

- Quantifies risk
- Derives the expected rate of return for a portfolio of assets and an expected risk measure
- Shows that the variance of the rate of return is a meaningful measure of portfolio risk
- Derives the formula for computing the variance of a portfolio, showing how to effectively diversify a portfolio
- Provides both:
  - the foundation for Modern Finance
  - a key tool for Haugen’s New Finance

### Assumptions of Markowitz Portfolio Theory

1. Investors consider each investment alternative as being presented by a probability distribution of expected returns over some holding period.

### Assumptions of Markowitz Portfolio Theory

2. Investors minimize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth.
  - I.e., investors like higher returns, but they are risk-averse in seeking those returns
  - And, again, this is a one-period model (i.e., the portfolio will need to be rebalanced at some point in the future in order to remain optimal)

### Assumptions of Markowitz Portfolio Theory

3. Investors estimate the risk of the portfolio on the basis of the variability of expected returns.
  - I.e., out of all the possible measures, variance is the key measure of risk

### Assumptions of Markowitz Portfolio Theory

4. Investors base decisions solely on expected return and risk, so their utility curves are a function of only expected portfolio returns and the expected variance (or standard deviation) of portfolio returns.
  - Investors' utility curves are functions of only expected return and the variance (or standard deviation) of returns.
  - Stocks' returns are normally distributed or follow some other distribution that is fully described by mean and variance.

### Assumptions of Markowitz Portfolio Theory

5. For a given risk level, investors prefer higher returns to lower returns. Similarly, for a given level of expected returns, investors prefer less risk to more risk.

### Markowitz Portfolio Theory

Using these five assumptions, a single asset or portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk, or lower risk with the same (or higher) expected return.

## Expected Rates of Return

- For an individual asset:
  - sum of the potential returns multiplied with the corresponding probability of the returns
- For a portfolio of assets:
  - weighted average of the expected rates of return for the individual investments in the portfolio

## Computation of Expected Return for an Individual Risky Investment

Exhibit 7.1

Probability	Possible Rate of Return (Percent)	Expected Return (Percent)
0.25	0.08	0.0200
0.25	0.10	0.0250
0.25	0.12	0.0300
0.25	0.14	0.0350
		E(R) = 0.1100

$$E(R_i) = \sum_{s=1}^n R_{i,s} P_s$$

where:

$P_s$  = the probability of states occurring

$R_{i,s}$  = the return on stock  $i$  in states

## Computation of the Expected Return for a Portfolio of Risky Assets

Weight ( $W_i$ ) (Percent of Portfolio)	Expected Security Return ( $R_i$ )	Expected Portfolio Return ( $W_i \times R_i$ )
0.20	0.10	0.0200
0.30	0.11	0.0330
0.30	0.12	0.0360
0.20	0.13	0.0260
		E( $R_{\text{port}}$ ) = 0.1150

$$E(R_{\text{port}}) = \sum_{i=1}^n W_i E(R_i)$$

Exhibit 7.2

where :

$W_i$  = the percent of the portfolio in asset  $i$

$E(R_i)$  = the expected rate of return for asset  $i$

## Variance (Standard Deviation) of Returns for an Individual Investment

Variance is a measure of the variation of possible rates of return  $R_i$ , away from the expected rate of return  $[E(R_i)]$ .

Standard deviation is the square root of the variance.

## Variance and Standard Deviation of Returns for an Individual Investment

Formulas:

## Variance (Standard Deviation) of Returns for an Individual Investment

Exhibit 7.3

Possible Rate of Return ( $R_i$ )	Expected Return $E(R_i)$	$R_i - E(R_i)$	$[R_i - E(R_i)]^2$	$P_i$	$[R_i - E(R_i)]^2 P_i$
0.08	0.11	0.03	0.0009	0.25	0.000225
0.10	0.11	0.01	0.0001	0.25	0.000025
0.12	0.11	0.01	0.0001	0.25	0.000025
0.14	0.11	0.03	0.0009	0.25	0.000225
					0.000500

Variance ( $\sigma^2$ ) = .0050

Standard Deviation ( $\sigma$ ) = .02236

## Standard Deviation of Returns for a Portfolio

Formula:

Two-Stock:

More than two stocks:

## Portfolio Standard Deviation Calculation

- Any asset of a portfolio may be described by two characteristics:
  - The expected rate of return
  - The expected standard deviations of returns
- A third characteristic, the covariance between a pair of stocks, also drives the portfolio standard deviation
  - Unlike portfolio expected return, portfolio standard deviation is not simply a weighted average of the standard deviations for the individual stocks
  - For a well-diversified portfolio, the main source of portfolio risk is covariance risk; the lower the covariance risk, the lower the total portfolio risk

## Covariance of Returns

- Covariance is a measure of:
  - the degree of “co-movement” between two stocks’ returns, or
  - the extent to which the two variables “move together” relative to their individual mean values over time

## Covariance of Returns

For two assets, i and j, the covariance of rates of return is defined as:

$$\sigma_{ij} = \int (R_i - E(R_i))(R_j - E(R_j))dP_{i,j}$$

or

$$\sigma_{ij} = \sum_{s \in S} (R_{is} - E(R_i))(R_{js} - E(R_j))P_s$$

## Covariance and Correlation

- The correlation coefficient is obtained by standardizing (dividing) the covariance by the product of the individual standard deviations

## Covariance and Correlation

Correlation coefficient varies from -1 to +1

$$r_{ij} = \frac{\text{Cov}_{ij}}{\sigma_i \sigma_j}$$

where:

$r_{ij}$  = the correlation coefficient of returns

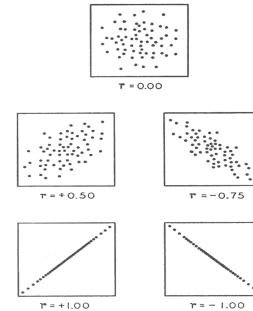
$\sigma_i$  = the standard deviation of  $R_{it}$

$\sigma_j$  = the standard deviation of  $R_{jt}$

## Correlation Coefficient

- It can vary only in the range +1 to -1. A value of +1 would indicate perfect positive correlation. This means that returns for the two assets move together in a completely linear manner. A value of -1 would indicate perfect correlation. This means that the returns for two assets have the same percentage movement, but in opposite directions

## Correlation Coefficient



## Parameters vs. Estimates

- Unfortunately, no one knows the *true* values for the expected return and variance and covariance of returns
- These must be estimated from the available data
- The most basic way to estimate these is the *naïve* or *unconditional* estimate
  - uses the sample mean, sample variance, and sample covariance from a time series sample of stock returns
  - Typical time series used is last 60 months' (5 years') worth of monthly returns
  - More sophisticated methods for estimating these will be discussed in subsequent chapters

## Estimation of Average Monthly Returns for Coca-Cola and Home Depot: 2001 Exhibit 7.4

### Computation of Monthly Rates of Return

Date	Closing Price	Dividend	Return (%)	Closing Price	Dividend	Return (%)
Dec.00	60.938			45.688		
Jan.01	58.000		-4.82%	48.200		5.50%
Feb.01	53.030		-8.57%	42.500		-11.83%
Mar.01	45.160	0.18	-14.50%	43.100	0.04	1.51%
Apr.01	46.190		2.28%	47.100		9.28%
May.01	47.400		2.62%	49.290		4.65%
Jun.01	45.000	0.18	-4.68%	47.240	0.04	-4.08%
Jul.01	44.600		-0.89%	50.370		6.63%
Aug.01	48.670		9.13%	45.950	0.04	-8.70%
Sep.01	46.850	0.18	-3.37%	38.370		-16.50%
Oct.01	47.880		2.20%	38.230		-0.36%
Nov.01	46.960	0.18	-1.55%	46.650	0.05	22.16%
Dec.01	47.150		0.40%	51.010		9.35%

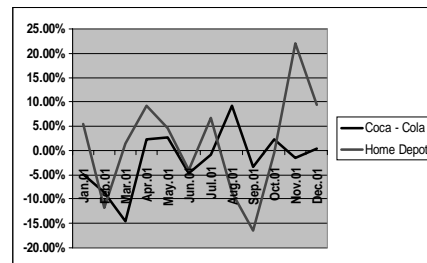
E(RCoca-Cola) = -1.81%      E(Rhome Depot) = 1.47%

## Estimation of Standard Deviation of Returns for Coca-Cola and Home Depot: 2001

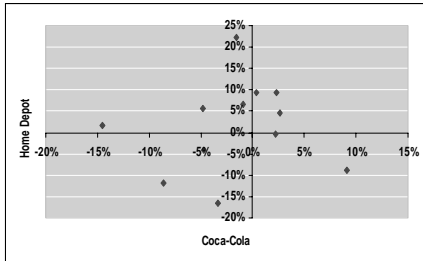
Date	Coca-Cola		Home Depot	
	$R_i - E(R_i)$	$[R_i - E(R_i)]^2$	$R_j - E(R_j)$	$[R_j - E(R_j)]^2$
Jan.01	-3.01%	0.000905	4.03%	0.001625
Feb.01	-6.76%	0.004565	-13.30%	0.017680
Mar.01	-12.69%	0.016100	0.04%	0.000000
Apr.01	4.09%	0.001675	7.81%	0.006106
May.01	4.43%	0.001964	3.18%	0.001013
Jun.01	-2.87%	0.000824	-5.54%	0.003074
Jul.01	0.92%	0.000085	-5.16%	0.002662
Aug.01	10.94%	0.011964	-10.16%	0.010327
Sep.01	-1.56%	0.000242	-17.96%	0.032266
Oct.01	4.01%	0.001609	-1.83%	0.000335
Nov.01	0.27%	0.000007	20.69%	0.042803
Dec.01	2.22%	0.000492	7.88%	0.006209
	0.040434		0.124101	

Variance<sub>C</sub> = 0.040434 / 12 = 0.003370      Variance<sub>H</sub> = 0.124101 / 12 = 0.010342  
 Standard Deviation<sub>C</sub> = 0.003370<sup>1/2</sup> = 0.0580      Standard Deviation<sub>H</sub> = 0.010342<sup>1/2</sup> = 0.1017

## Times Series Returns for Coca-Cola and Home Depot: 2001



### Scatter Plot of Monthly Returns for Coca-Cola and Home Depot: 2001



### Estimation of Covariance of Returns for Coca-Cola and Home Depot: 2001

Date	Return (%)	Return (%)	Ri - E(Ri)	Rj - E(Rj)	[Ri - E(Ri)] X [Rj - E(Rj)]
Jan 01	-4.82%	5.50%	-3.01%	4.03%	-0.001213
Feb 01	-8.57%	-11.83%	-6.76%	-13.30%	0.000984
Mar 01	-14.50%	1.51%	-12.69%	0.04%	-0.000050
Apr 01	2.28%	9.28%	4.09%	7.81%	0.003199
May 01	2.62%	4.65%	4.43%	3.18%	0.001411
Jun 01	-4.68%	4.08%	-2.87%	-5.54%	0.001592
Jul 01	-0.89%	6.63%	0.92%	5.16%	0.000477
Aug 01	9.13%	-8.70%	10.94%	-10.16%	-0.011115
Sep 01	-3.37%	-16.50%	-1.56%	-17.96%	0.002797
Oct 01	2.20%	-0.36%	4.01%	-1.83%	-0.000735
Nov 01	-1.55%	22.16%	0.27%	20.69%	0.000552
Dec 01	0.40%	9.35%	2.22%	7.88%	0.001747
E(RCoca-Cola) =	-1.81%	E(RHomeDepot) =	1.47%		
Covij =	0.007645			/ 12 =	0.000637
Corij =	Covij / (stdev(i) * stdev(j)) =				0.1079
				Sum	0.007645

### Combining Stocks with Different Returns and Risk

- Assets may differ in expected rates of return and individual standard deviations
- Negative correlation reduces portfolio risk
- Combining two assets with -1.0 correlation reduces the portfolio standard deviation to zero only when individual standard deviations are equal

### Combining Stocks with Different Returns and Risk

Asset	E(R <sub>i</sub> )	W <sub>i</sub>	σ <sup>2</sup> <sub>i</sub>	σ <sub>i</sub>
1	.10	.50	.0049	.07
2	.20	.50	.0100	.10

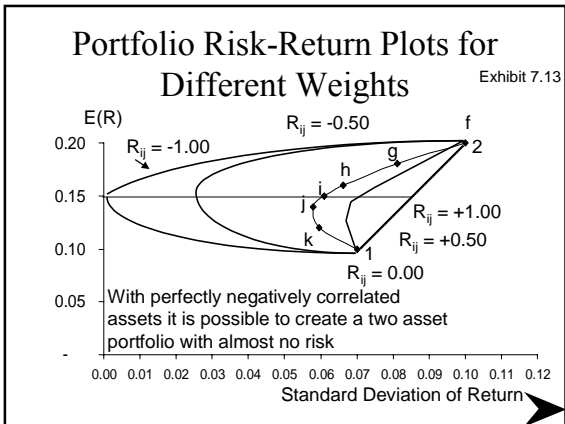
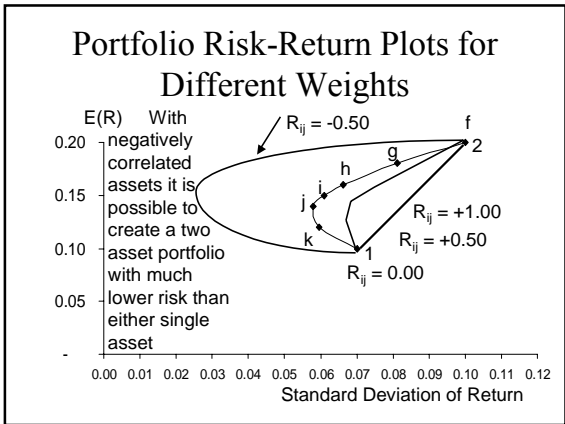
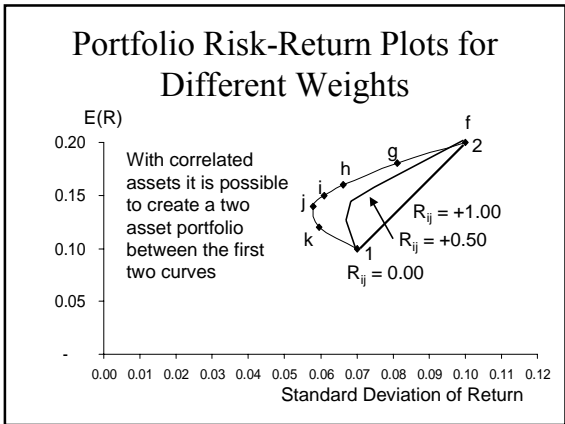
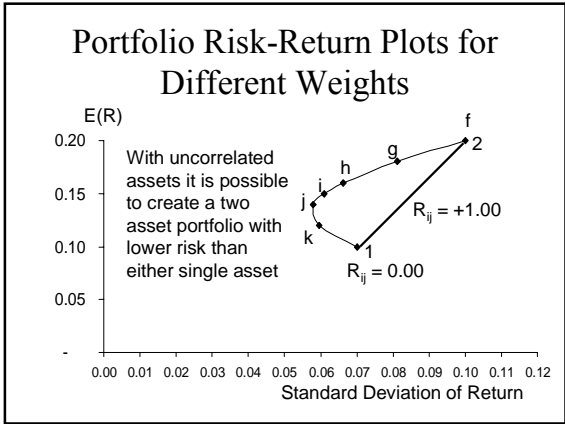
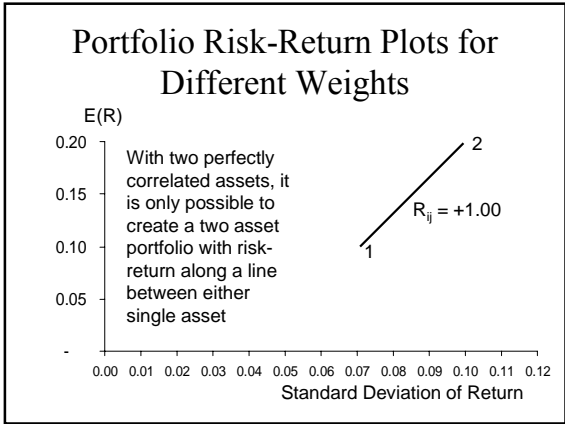
Case	Correlation Coefficient	Covariance
a	+1.00	.0070
b	+0.50	.0035
c	0.00	.0000
d	-0.50	-.0035
e	-1.00	-.0070

### Constant Correlation with Changing Weights

Asset	E(R <sub>i</sub> )	r <sub>ij</sub> = 0.00	
Case	W <sub>1</sub>	W <sub>2</sub>	E(R <sub>i</sub> )
f	0.00	1.00	0.20
g	0.20	0.80	0.18
h	0.40	0.60	0.16
i	0.50	0.50	0.15
j	0.60	0.40	0.14
k	0.80	0.20	0.12
l	1.00	0.00	0.10

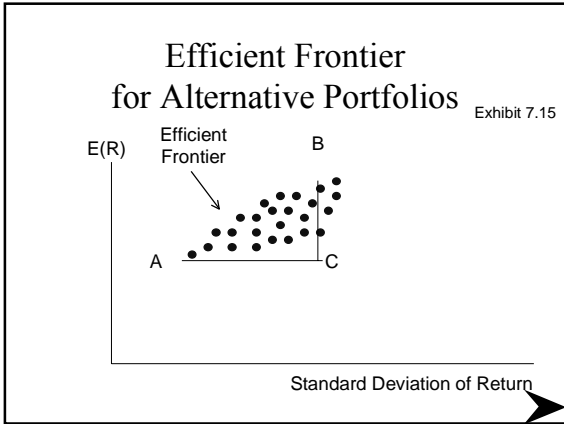
### Constant Correlation with Changing Weights

Case	W <sub>1</sub>	W <sub>2</sub>	E(R <sub>i</sub> )	E(σ <sub>port</sub> )
f	0.00	1.00	0.20	0.1000
g	0.20	0.80	0.18	0.0812
h	0.40	0.60	0.16	0.0662
i	0.50	0.50	0.15	0.0610
j	0.60	0.40	0.14	0.0580
k	0.80	0.20	0.12	0.0595
l	1.00	0.00	0.10	0.0700



### The Efficient Frontier

- The efficient frontier represents that set of portfolios with the maximum rate of return for every given level of risk, or the minimum risk for every level of return
- Frontier will be portfolios of investments rather than individual securities
  - Exceptions being the asset with the highest return and the asset with the lowest risk

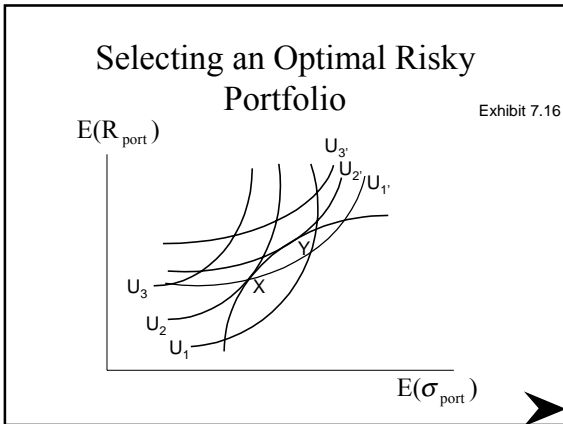


### The Efficient Frontier and Investor Utility

- An individual investor's utility curve specifies the trade-offs he is willing to make between expected return and risk
- The slope of the efficient frontier curve decreases steadily as you move upward
- These two interactions will determine the particular portfolio selected by an individual investor

### The Efficient Frontier and Investor Utility

- The optimal portfolio has the highest utility for a given investor
- It lies at the point of tangency between the efficient frontier and the utility curve with the highest possible utility



### Estimation Issues

- Results of portfolio allocation depend on accurate statistical inputs
- Estimates of
  - Expected returns ( $n$  estimates)
  - Standard deviation ( $n$  estimates)
  - Correlation coefficient ( $[n(n-1)/2]$  estimates)
    - Among entire set of assets
    - With 100 assets, 4,950 correlation estimates
    - With 500 assets, 124,750 correlation estimates
- Estimation risk refers to potential errors
  - Typically only have between  $60n$  and  $260n$  observation data points from which to obtain estimates

### Estimation Issues

- With assumption that stock returns can be described by a single market model, the number of correlation inputs required reduces to the number of assets, plus one
- Single index market model:
 
$$R_{it} = a_i + b_i R_{mt} + \epsilon_{it}$$

$b_i$  = the slope coefficient that relates the returns for security  $i$  to the returns for the aggregate stock market

$R_{mt}$  = the return for the aggregate stock market during time period  $t$

## Estimation Issues

If all the securities are similarly related to the market and a  $b_i$  derived for each one, it can be shown that the correlation coefficient between two securities  $i$  and  $j$  is given as:

$$r_{ij} = b_i b_j \frac{\sigma_m^2}{\sigma_i \sigma_j}$$

where  $\sigma_m^2$  = the variance of returns for the aggregate stock market

## Implementation Issues

- If portfolios on the efficient frontier are *optimal*, why don't all investors use Markowitz portfolio optimization?
- Three key problems complicate implementation:
  1. Too many inputs required
  2. Use of estimates can lead to "error maximization"
  3. Reliance on historical data to obtain estimates

## Implementation Issues

1. Too many inputs required
  - Limit use to asset allocation or small-scale problems
  - Use factor models to obtain / develop correlation estimates
    - Allows for use with much larger scale problems
    - E.g., all 1700 stocks that Value Line follows
    - Simplest factor model is the "single-index market model"

## Implementation Issues

2. Use of estimates can lead to "error maximization"
  - Introduce additional "hard" constraints in optimization process
    - E.g., Haugen, p. ??
    - Use *stratified sampling*
    - Precludes optimized portfolio from being too different from the benchmark portfolio
  - Use "portfolio resampling" to find *average* optimal portfolio given range of possible estimates

## Implementation Issues

3. Reliance on historical data to obtain estimates
  - True parameters not only never known, but also not constant over time
  - Factor models and stratified sampling help here, too
    - Factor model relationships tend to be more stable than relationships between individual stocks
    - Stratified sampling constraints prevent portfolio from falling too far behind benchmark even when estimated relationships change

## Minimum Variance Portfolio (MVP)

1. Two-stock case:

$$\sigma_{\text{port}} = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}_{12}}$$

2. Multiple stock case:

$$\sigma_{\text{port}} = \sqrt{\sum_i w_i^2 \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{Cov}_{ij}} = \sqrt{w' \Sigma w}$$

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Future topics  
Chapter 8

- Capital Market Theory
- Capital Asset Pricing Model
- Beta
- Expected Return and Risk
- Arbitrage Pricing Theory

