On Limitations of PERT Methodology for Managing Large Scale Projects[1]

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Abstract

The PERT based Project Management tools do not take an adequate account of risk. Monte Carlo Simulation provides an improved methodology to quantify schedule risk. This paper investigates the use of this methodology and its shortcomings in an Engineering environment for better understanding, designing, and managing a project such that the detrimental effects of these risks can be minimized.

Introduction

- PERT is traditionally used for 'non-routine' projects – its origin is in the design and development of Nautilus submarine http://www.ussnautilus.org/.
- on the other hand CPM is for routine projects, like "maintenance operations at DuPont" http://ieeexplore.ieee.org/iel2/682/5877/00225305.pdf?arnumber=225305
- Large-scale engineering projects are usually original, one-of-a-kind projects. At its current state, PERT is not fully capable of handling all the features of such projects:
    - PERT is designed to handle only the so-called "PERT-distribution" for random activity durations with the three estimates for activity durations. However, in many projects there is a need for other probability distributions, as well as "activities with selection probabilities."

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- PERT assumes that there exists “a single dominant critical path” in the project network, which most likely not true in most very-large scale engineering projects,
- PERT assumes that random variables representing the task times on the critical path are “independently and identically distributed” (IID) random variables when making its conclusions as to the completion probabilities of the project, which most likely not true in most very-large scale engineering projects.
- This paper emphasized these issues and proposes some suggestions.

Technical considerations

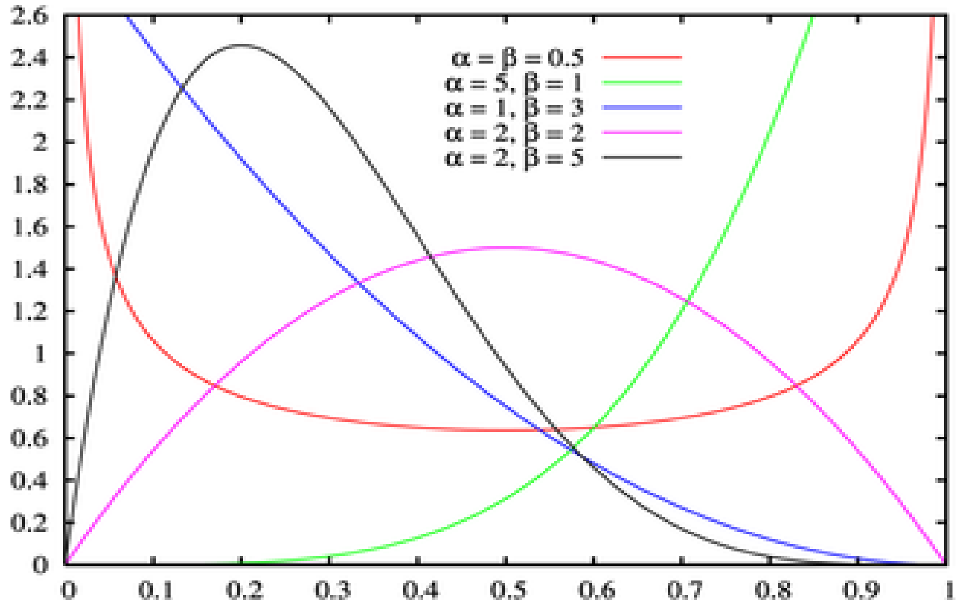
On Beta Distribution

The risk of failing to complete activities and entire projects on time and the resulting cost overruns are critical elements of all operations in this project. The PERT method introduces uncertainty into the network by treating each activity’s completion time as a random variable. The probability distribution of the activity time random variable is almost universally taken to be “beta” distribution. Its distribution function called the ***beta function***:

$$B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}.$$

A random variable X has the beta distribution, if its density function is given by the following

$$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1},\ 0 < x < 1.$$



The two parameters α and β are shape parameters and hence produce a family of distributions. The triangular distribution as well as the uniform distribution, as assumed in CPM (‘critical path method’) networks, are special cases of beta distribution, and normally used to avoid dealing with the mathematical difficulties arising in the analysis under the Beta distribution.. The expected value and standard deviation of beta distribution is used in PERT systems as a standard practice. Special properties of beta distribution will be used as discussed below to arrive at analysis that is more reliable.

Statistic	Standard beta	General beta
Range	$(0,1)$	(A,B)
Mean	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha B+\beta A}{\alpha+\beta}$
Mode $\alpha,\beta>1$	$\frac{\alpha-1}{\alpha+\beta-2}$	$\frac{B(\alpha-1)+A(\beta-1)}{\alpha+\beta-2}$
Variance	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$\frac{\alpha\beta(B-A)^2}{(\alpha+\beta)^2(\alpha+\beta+1)}$

One needs to focus on individual task completion times, examining the probability distribution generally used to model the uncertainty in activity completion times in simulation and in PERT setting. One must also examine the common assumptions used to derive estimates of the parameters required to describe the beta distribution and to derive mean and standard deviation of project completion times in the underlying PERT representation. It is also worth mentioning that PERT approximations of project duration and the associated probability statements derived from those approximations may sometimes be misleading. The most widely recognized source of bias is near critical paths that emerge as critical in a realization of the project. However, simulation can overcome the bias in the PERT and in particular in standard deviation of completion times by bypassing the PERT formulas. This requires more careful choices parameters of the beta distribution so that a wider variety of shapes can be used to model activity completion times in a network simulation.

The formal beta distribution used in many simulation software packages such as EXTEND, CORE, and in MS EXCEL is a continuous distribution that has several distinct properties:

1. The beta has finite limits. Many real-world random variables, including activity completion times, have finite limits. For example, task completion times have a lower bound that is greater than zero because it is impossible to complete a task in less than zero time, and this should be reflected in the model. By contrast, the normal distribution has a wider range from negative infinity to plus infinity.

2. The beta can be asymmetrical. This property is desirable for modeling activity completion times, which are often skewed to the right by unlikely but severe overruns.

3. Finally, the beta distribution is flexible because it can take on many different shapes, including flat, narrow, U and inverted-U shapes.

These properties along with others would lay a foundation for variance reduction analysis of the project's completion time. Such analysis could be of a great value in terms of providing the decision makers with some degree of flexibilities in making critical decisions.

On Budgeting Issues

Cleary, all projects have to be supported financially. The budget allocated to the project, however, is subject to uncertainty due to various financial, market, and political risks. Thus, an important piece of the analysis is to incorporate budget uncertainty into project time-cost tradeoff. In a realistic model, one formulates financial feasibility as a stochastic constraint, transforms it into a deterministic equivalent in the case of normal, beta, or triangular distribution, and solves the equivalent model accordingly. Typical result is a minimum time-cost curve, which relates the shortest project duration to different levels of budget and available resources. An important assessment is to show the degree to which budget uncertainty relates to financial constraint and, thus, provide extra contingency duration when necessary. For example, if the financial constraint has to be met at a higher probability level, extra contingency costs are necessary to ensure an on-time completion.

At the activity level, a more elaborate budget analysis may be incorporated. This is commonly known as "Project Crashing" or "Crashing and Time/Cost Tradeoffs" in which budget and generally resource allocation alternatives are analyzed in terms of their incremental effects on the completion time of individual activities and on the overall project's completion time.

There are three basic cost categories involved in most projects - direct costs, indirect costs, and penalty costs. Direct costs include labor, material, and other costs directly related to completing activities. Activity times may be shortened by applying additional resources such as overtime, special equipment, or more personnel. As additional resources are added, the direct costs increase. Indirect costs are the overhead costs that support the project, such as administration, financial, security, maintenance, and depreciation. Shortening the project time may decrease these costs. The shorter the project, the lower the indirect costs. Additionally, a project may also involve penalty costs; these are incurred if the project extends beyond a specific date. Generally, because of complication in solving underlying models under crashing, heuristic approaches are used. The minimum project duration under crashing phenomenon is not part of the simulation tools and must be incorporated separately. Typically, optimization modeling tools are used to make more reliable estimates of the minimum project's completion time under crashing. Software tools dealing with PERT systems or those capable of carrying out crashing analysis, would normally assume a linear, or convex, activity cost function. This implies, for example, the cost of performing an activity is a linear or convex function of its duration. Quite the contrary, in a realistic setting, the cost is a concave function of the input parameter (in this case duration). The reason for the concavity of the cost function is the presence of the “economies of scale” in the process. Clearly, the rate of cost increase would be decreasing as the duration increases. This simple fact, and incorporation of its effects in the model network, cannot be easily captured. This is because the overall project cost would be a concave function. As a result of this, a theoretical complexity is added to the overall model, making it extremely difficult to solve using available software tools. A specialized set of tools and expertise would be needed to solve the model.

This latter area of research is known as “Global Optimization” and currently is the frontier of theoretical research in mathematical optimization. As a consequence, a more realistic estimation of the project’s completion time and further assessments on time-cost trade-off require a more comprehensive analysis not provided in the commercially available simulation software packages and conventional tools.

Conclusions

Schedule simulation should be used on any large or complex Engineering project since traditional mathematical analysis technique such as the Critical Path Method (CPM) and the Program Evaluation and Review Technique (PERT) do not account for path convergence and thus tend to underestimate project duration. It is easier to demonstrate, with a simulation, how task(s) not on the critical path may end up on the critical path due to deviations from the plan and derail a project. Also, with simulation one can illustrate the negative impact of parallel paths converging at critical points. This paper was geared toward this end.

References

- Galway, L., “Quantitative Risk Analysis for Project Management: A Critical Review,” WR-112-RC, Rand Corporation, February 2004.