PROCEEDINGS

OF THE

THIRTY-SECOND ANNUAL MEETING

OF THE

WESTERN DECISION SCIENCES INSTITUTE

April 15 - 19, 2003

Marriott Resort and Beach Club
Island of Kauai, Hawaii

Kauai 2003

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ISSN 1098-2248

LOGIC-BASED MODELS FOR A CLASS OF LOT STREAMING PROBLEMS

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ABSTRACT

The models presented in this study are based on lot streaming paradigm and make use of the basic modeling framework of constraint satisfaction. The problem modeled is the following: Several production lots, each consisting of a number of units, are to be processed in a job shop. Each production lot, or its sublot of given size, has a deadline. The problem is to determine the timing and size of transfer batches subject to inventory balance equations and resource constraints.

INTRODUCTION

In the past, production scheduling problem was not "...a visible one in many firms because other parts of the firm have absorbed much of the impact of poor scheduling." [8] It is no longer the case. Changing nature of business competitiveness demands highly advanced production scheduling systems. Such systems, to be viable, must be computationally feasible and be able to handle the intricacies of modern manufacturing systems. Current developments in information and decision technologies make it possible to design such systems.

Production planning problems customarily are posed as periodic review processes. Detailed scheduling problems on the other hand, extending over relatively shorter planning horizons, require continuous time domain.

A major problem of production planning is how to handle sequencing requirements on resources, whereas machine scheduling cannot handle lot sizing. *Lot streaming* may provide the necessary conceptual framework for integrating lot sizing and machine scheduling [3]. Lot streaming basically is moving some portion of a process batch ahead to begin a downstream operation. Classical machine scheduling theory envision an operation as an elemental task to be performed. It is assumed that "[t]he processing times of successive operations of a particular job cannot be overlapped. A job can be in process on at most one operation at a time" [4]. This assumption is justified when jobs are monolithic entities. But in the case of scheduling production lots, each consisting of a number of units, it may be overly restrictive. The processing time of such a lot is comprised of a (usually "detached") setup time and the sum of the processing times of each unit in the lot. For instance, when the machine is available, it is not reasonable to delay its setup until *all* the items arrive from the upstream machine.

Lot streaming, in this context, was introduced in papers by Baker [2] and Trietsch [10]. In a later joint work [11], they discuss the practical importance of this approach. A number of manufacturing management innovations, such as Group Technology (leading to cell based manufacturing, resulting in shorter lead times and reduced work in progress inventories), Just-in-Time Systems ("lot size of one"), and OPT/Synchronous Manufacturing (transfer *vs.* process batches) paved the way for lot streaming theory which attempts rigorous analytical treatment of these issues. In the recent years lot streaming attracted considerable attention in the machine scheduling community. Most of the results are on streaming a single job in a flow shop with a makespan objective [6]. Results on multiple jobs are more limited: Dauzère-Pérès and Lasserre [5] analyzed job shops; and open shops were analyzed in Sen and Benli [9].

In this paper an event-time model for scheduling production is presented. It is based on lot streaming paradigm and makes use of the basic modeling framework of constraint satisfaction [7]. Events, in this model, are ordered sequence of material handling moves (interstage material transfers) in a job shop. There are two types of events:

exogenous events whose time of occurrence are given (as parameters of the problem), such as demand occurrences, order due dates or deadlines, and

endogenous events whose occurrence times are decision variables of the model, such as interstage material (WIP) movements.

THE PROBLEM

The problem to be considered in this paper is the following: Several jobs (*"production lots"*), each consisting of a number of units, are to be processed in a job shop. At each stage, there can be identical parallel machines and jobs may require additional resources besides the machine on which they are processed. Total processing time of a job at a stage is the sum of processing times of (identical) units comprising the production lot. It is assumed that any setup time, if it exists, is negligible.
A unit of the production lot that is being processed at a particular machine is referred to as an *item* and the corresponding operation is referred to as a *stage*. Among the items making up a job, there is demand interaction in the form of supply-demand relationship ("in order to produce one item you need another item as an input".) Thus each job can be viewed as a serial manufacturing process. The last item in this serial process is a unit of the end product. For example, the raw material enters stage 1 (the first machine required by that job) in which item 1 is produced. Item 1 goes into stage 2 for the production of item 2, and so on, until the finished good, the end item, is produced in the last stage for that job. The amount and timing of demand for each end item is given and shortages are not allowed. Stating this in the terminology of machine scheduling: each production lot, or its sublot of given size, has a deadline.
The items are transferred in between stages in *sublots* by means of material handling equipment. There are limited number of such equipment at any given time. The problem is to determine the timing and size of transfer batches so as to optimize a given criterion, subject to inventory balance equations (in the form of input-output relationships, including the demand requirements of end items) and resource constraints. Various types of optimality criterion can be considered; such as minimizing total number of transfer batches, or other tardiness based measures. But in this paper we will be interested simply in finding a feasible solution.

A MATHEMATICAL PROGRAMMING APPROACH

Consider a multi-item periodic review model with *variable period lengths*. Let

$i \in \mathcal{I}$ be the index set of all items (or index set of all stages), and $i \in \mathcal{I}_e$ be the index set of end items. Also define

$\pi(i)$ as the predecessor of item i, and

$\sigma(i)$ as the successor of item i.

Inter-stage transfers can occur at time points $T_t,\ t \in \mathcal{T}$, where $\mathcal{T} = \{1, \ldots, n\}$ is the index set of all such time points. These time points are decision variables of the problem except for the points corresponding to exogenous events: the given deadlines of the end items. Let these time points be $\tau_1, \tau_2, \ldots, \tau_m$. Assume a reasonable "average" period length, say δ. Thus, there will be on average $(\tau_\ell - \tau_{\ell-1})/\delta$ periods in between two exogenous events at time points τ_ℓ and $\tau_{\ell-1}$. These event times are given by

$$T_{\nu_\ell} = \tau_\ell \ , \ \ell = 1, \ldots, m,$$

where

$$\nu_\ell = \sum_{j=1}^{\ell} \left\lceil \frac{\tau_j - \tau_{j-1}}{\delta} \right\rceil, \text{ and } \tau_0 \equiv 0.$$

Let a subset of $\mathcal{T} \supset \mathcal{T}_e$ denote set of time points of exogenous events corresponding to demand occurrences of end items.

A period with variable length is defined as the time interval $[T_{t-1}, T_t], t \in \mathcal{T}$, and referred to as period t.

At each stage, i, consider two *buffers*. During period t the material to be processed, $\pi(i)$, stored in the *input buffer* and the processed material (item i), that is not transferred to the predecessor stage $\sigma(i)$, is stored in the *output buffer*.

We can now define the following decision variables:

X_{it} is the amount of item i processed during $[T_{t-1}, T_t]$, i.e. amount of processing in period t in stage i, and

L_{it} is amount of item i made available for the production of item $\sigma(i)$ at time T_t, i.e. the size of the transfer batch at the end of period t in stage i.

In order to achieve the inventory balance, we need to define two sets of inventory variables:

I_{it} is the input inventory level (amount in the input buffer at T_t) of item i at the end of period t, i.e. amount of production in period t in stage i, and

O_{it} is output inventory level (amount in the output buffer at T_t) of item i at the end of period t.

In order to indicate whether or not a transfer takes place at stage i at the end of period t, define a set of binary variables,

$$Y_{it} = \begin{cases} 1, & \text{if there is a transfer batch from stage } i \text{ to stage } \sigma(i) \text{ at the end of period } t, \\ 0, & \text{otherwise.} \end{cases}$$

Finally, let

$k \in \mathcal{K}$ be the index set of resources used by all items,

ρ_{ki} be the amount of resource $k \in \mathcal{K}$ required for processing a unit of item i, and

r_k be the total availability of resource k per unit time.

There are two sets of *inventory balance equations*, for input and output buffers, respectively:

$$I_{i,t-1} + L_{\pi(i),t-1} = I_{it} + X_{it},\ \forall\ i,t, \tag{1}$$
$$O_{i,t-1} + X_{it} = O_{it} + L_{it},\ \forall\ i,t. \tag{2}$$

Recall that demand for end items occur at time points $t \in \mathcal{T}_e$. Suppose the demand profile for end items is given as $\{d_{it}, i \in \mathcal{I}_e, t \in \mathcal{T}_e\}$, then in order to make sure that the demand for all end items met, it is sufficient to fix the corresponding transfer batch sizes as:

$$L_{it} = d_{it},\ i \in \mathcal{I}_e, t \in \mathcal{T}_e. \tag{3}$$

There are resource constraints restricting the amount that can be processed in a period,

$$\sum_i \rho_{ki}\ X_{it} \le r_k \left[T_t - T_{t-1}\right],\ \forall\ k,t. \tag{4}$$

The transfers can take place only if they are indicated to do so,

$$L_{it} \le MY_{it},\ \forall\ i,t, \tag{5}$$

where M is a very large number or transfer capacity, if applicable. Transfer times are assumed to be negligible. If these times are significant, then the inter-stage transfers can be assumed as a separate "stage" in processing with appropriate resource requirements.
In order to make sure that there is a material handling equipment available when required,

$$\sum_i Y_{it} \le \gamma,\ \forall\ t, \tag{6}$$

where γ is the number of material handling equipment available. If it takes a maximum of ϵ units of time for an interstage transfer to take place, then to ensure the availability of handling equipment at each period,

$$T_t \ge T_{t-1} + \epsilon,\ \forall\ t; \tag{7}$$

and with the appropriate nonnegativity, integrality, and the initial conditions.

A LOGIC-BASED APPROACH

The basic framework of logic-based modeling [7] may contribute to computational improvements, as well as improving certain aspects model's realism. The essence of this model is how the exogenous events whose times are given and the endogenous events whose times of occurrence are decision variables are handled in the same model. The mathematical programming approach presented above requires the assumption of estimating "a reasonable average period length" and then decides when the inter-stage transfers –the endogenous events– to take place within the predetermined fixed end points. Logic-based approach, on the other hand, can handle this aspect more realistically.
Let ν_ℓ denote the period at the end of which an exogenous event ℓ occurs. Recalling that τ_ℓ are the times at which those exogenous events occur, the *conditional constraint*

$$(\nu_\ell = t) \rightarrow (T_t = \tau_\ell), \quad t = 1,\ldots,n,$$

for each $\ell = 1, \ldots, m$, ensures that the exogenous event ℓ happens at T_t.
Considerable computational improvements may result in eliminating the big-M constraint, by more directly formulating as

$$(Y_{it} = 1) \rightarrow (L_{it} \geq 0)$$

$$(Y_{it} = 0) \rightarrow (L_{it} = 0)$$

Using the *defined predicate* "cumulative" in place of the resource constraints may further improve computational performance. Let I_k be the index set of items that require resource k. The rate of resource consumption for item $i \in I_k$ during period t is given by

$$\frac{\rho_{ki} X_{it}}{T_t - T_{t-1}},$$

where T_{t-1} is its start time and its duration is $T_t - T_{t-1}$. Letting T_k denote the start time of all items that use resource k, the constraint

$$\text{cumulative}\left((T), (T_t - T_{t-1}), \left(\frac{\rho_{ki} X_{it}}{T_t - T_{t-1}}\right), r_k\right)$$

for all k.

CONCLUSIONS

Constraint programming is a relatively new approach for solving combinatorial optimization problems. This approach is especially effective for large-scale problems with side conditions. In this paper modeling approaches of mathematical programming and logic-based methods are briefly compared and contrasted using a lot streaming problem. Until very recently there were no user-friendly software for constraint programming. But systems like OPL Studio [1], an integrated development environment for combinatorial optimization applications, include modules for constraint programming. OPL Studio has several tools including CPLEX (mathematical programming solver), SOLVER (solver for constraint programming) and SCHEDULER (a tool developed for constraint based scheduling). Once a model is constructed, after compilation, OPL Studio automatically detects the the problem type and determines the most convenient solver to solve it. Availability of such systems makes it possible to take advantage of vast modeling capabilities of logic-based methods in solving large-scale problems of the type discussed in this paper.

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