CONSTRAINT PROGRAMMING APPROACHES TO A LOT STREAMING PROBLEM OF MACHINE SCHEDULING

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Abstract: The event-time models presented in this study are based on lot streaming paradigm and make use of the basic modeling framework of constraint programming. The problem modeled is the following: Several production lots, each consisting of a number of units, are to be processed in a job shop. Each production lot, or its sublot of given size, has a deadline. The problem is to determine the timing and size of transfer batches subject to inventory balance equations and resource constraints.

Keywords: Constraint programming, lot streaming, machine scheduling, production planning.

Introduction

In the past, production scheduling problem was not "… a visible one in many firms because other parts of the firm have absorbed much of the impact of poor scheduling." (Pounds, 1961) It is no longer the case. Changing nature of business competitiveness demands highly advanced production scheduling systems. Such systems, to be viable, must be computationally feasible and be able to handle the intricacies of modern manufacturing systems. Current developments in information and decision technologies make it possible to design such systems.

Production planning problems customarily are posed as periodic review processes. Detailed scheduling problems on the other hand, extending over relatively shorter planning horizons, requiring continuous time domain. Among the first to comment on this incongruity was Uday Karmarkar (1994), comparing the different modeling perspectives in theory and practice; he stated that: "OR/MS [models] often treat capacity in terms of loading time buckets. However, in practice, it is much easier to think in terms of time lines and events and intervals … In some models, such as scheduling with Gantt charts, we use this kind of modeling, but we lack ways of dealing with decomposition or composition of these models. As a result, capacity and planning models are often formulated very differently. Perhaps this is one reason that time interval and release oriented methods like MRP and DRP are used in practice for planning even though they are completely unable to actually deal with resource allocation decisions." Likewise, Bill Maxwell (1997) criticized models that lump operations and events into large time buckets and proposed formulation of "event-time models" which are in essence "ordered sequence of time and data."

A major problem of production planning is how to handle sequencing requirements on resources, whereas machine scheduling cannot handle lot sizing. *Lot streaming* may provide the necessary conceptual framework for integrating lot sizing and machine scheduling (Benli, 2003). Lot streaming basically is moving some portion of a process batch ahead to begin a downstream operation. Classical machine scheduling theory envision an operation as an elemental task to be performed. It is assumed that "[t]he processing times of successive operations of a particular job cannot be overlapped. A job can be in process on at most one operation at a time" (Conway, Maxwell & Taylor, 1967). This assumption is justified when jobs are monolithic entities. But in the case of scheduling production lots, each consisting of a number of units, it may be overly restrictive. The processing time of such a lot is comprised of a (usually "detached") setup time and the sum of the processing times of each unit in the lot. For instance,

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when the machine is available, it is not reasonable to delay its setup until all the items arrive from the upstream machine.

Lot streaming, in this context, was introduced in papers by Baker (1988) and Trietsch (1987). In a later joint work (Trietsch & Baker, 1993), they discuss the practical importance of this approach. A number of manufacturing management innovations, such as Group Technology (leading to cell based manufacturing, resulting in shorter lead times and reduced work in progress inventories), Just-in-Time Systems ("lot size of one"), and OPT/Synchronous Manufacturing (transfer vs. process batches) paved the way for lot streaming theory which attempts rigorous analytical treatment of these issues. In the recent years lot streaming attracted considerable attention in the machine scheduling community. Most of the results are on streaming a single job in a flow shop with a makespan objective (Glass & Potts, 1998). Other criteria are analyzed in (Sen, Benli & Topaloglu, 1998). Results on multiple jobs are more limited: Dauzere-Peres and Lasserre (1997) analyzed job shops; and open shops were analyzed in Sen and Benli (1999). For a comprehensive review of recent research on lot streaming see Chang and Chiu (2005).

In this paper an event-time model for scheduling production is presented. It is based on lot streaming paradigm and makes use of the basic modeling framework of constraint programming (Hooker, 2000). Events, in this model, are ordered sequence of material handling moves (interstage material transfers) in a job shop. There are two types of events:

- *exogenous* events whose time of occurrence are given (as parameters of the problem), such as demand occurrences, order due dates or deadlines, and
- *endogenous* events whose occurrence times are decision variables of the model, such as interstage material WIP movements.

The Problem

The problem to be considered in this paper is the following: Several jobs ("production lots"), each consisting of a number of units, are to be processed in a job shop. At each stage, there can be identical parallel machines and jobs may require additional resources besides the machine on which they are processed. Total processing time of a job at a stage is the sum of processing times of (identical) units comprising the production lot. It is assumed that any setup time, if it exists, is negligible.

A unit of the production lot that is being processed at a particular machine is referred to as an *item* and the corresponding operation is referred to as a *stage*. Among the items making up a job, there is demand interaction in the form of supply-demand relationship (``in order to produce one item you need another item as an input".) Thus each job can be viewed as a serial manufacturing process, as shown Figure 1.

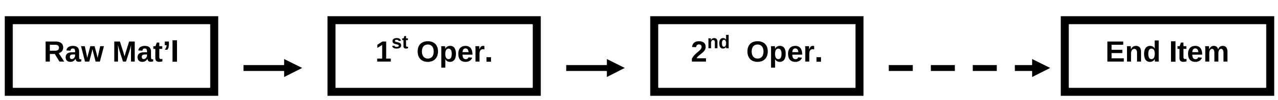

Figure 1. Serial Manufacturing Process

The last item in this serial process is a unit of the end product. For example, the raw material enters stage 1 (the first machine required by that job) in which item 1 is produced. Item 1 goes into stage 2 for the production of item 2, and so on, until the finished good, the end item, is produced in the last stage for that job. The amount and timing of demand for each end item is given and shortages are not allowed. Stating this in the terminology of machine scheduling: each production lot, or its sublot of given size, has a *deadline*.

The items are transferred between stages in *sublots* by means of material handling equipment. There are limited number of such equipment at any given time. The problem is to determine the timing and size of transfer batches so as to optimize a given criterion, subject to inventory balance equations (in the form of input-output relationships, including the demand requirements of end items) and resource constraints. Various types of optimality criterion can be considered; such as minimizing total number of transfer batches or other tardiness based measures. But in this paper we will be interested simply in finding a feasible solution.

The Model

Consider a multi-item periodic review model with "variable period lengths". Let

- $i \in I$ be the index set of all items (or index set of all stages), and $i \in I_e$ be the index set of end items. Also define
- $\pi(i)$ as the predecessor of item *i*, and
- $\sigma(i)$ as the successor of item *i*.

Inter-stage transfers can occur at time points $\mathrm{T}_t, t \in T$, where $T = \{1, \cdots, n\}$ is the index set of all such time points. These time points are decision variables of the problem except for the points corresponding to exogenous events: the given deadlines of the end items. Let these time points be $\tau_1, \cdots, \tau_m$. Let a subset of $T \supset T_e$ denote set of time points of exogenous events corresponding to demand occurrences of end items. A period with variable length is defined as the time interval $[\mathrm{T}_{t-1}, \mathrm{T}_{\mathrm{t}}], t \in T,$ and referred to as period *t*. At each stage, *i*, consider two "buffers". During period *t*, the material to be processed, $\pi(i)$, stored in the "input buffer" and the processed material (item *i*), that is not transferred to the predecessor stage $\sigma(i)$ is stored in the "output buffer".

We can now define the following decision variables:

- X_{it} is the amount of item *i* processed during $[\mathrm{T}_{t-1}, \mathrm{T}_{\mathrm{t}}],$ i.e. amount of processing in period *t* in stage *i*, and
- L_{it} is amount of item *i* made available for the production of item $\sigma(i)$ at time T_{t}, i.e. the size of the transfer batch at the end of period *t* in stage *i*.

In order to achieve the inventory balance, we need to define two sets of inventory variables:

- I_{it} is the input inventory level (amount in the input buffer at T_{t}) of item *i* at the end of period *t*, i.e. amount of production in period *t* in stage *i*, and
- O_{it} is output inventory level (amount in the output buffer at T_{t}) of item *i* at the end of period *t*.

In order to indicate whether or not a transfer takes place at stage *i* at the end of period *t*, define a set of binary variables,

- $Y_{it} = 1$, if there is a transfer batch from stage *i* to stage $\sigma(i)$ at the end of period *t*, and
- $Y_{it} = 0$, otherwise.

Finally, let

- $k \in K$ be the index set of resources used by all items,
- ρ_{ki} be the amount of resource $k \in K$ required for processing a unit of item *i*, and
- r_k be the total availability of resource *k* per unit time.

There are two sets of *inventory balance equations*, one for input and output buffers each:

$$I_{i,t-1} + L_{\pi(i),t-1} = I_{it} + X_{it}, \forall i,t, \tag{1}$$
$$O_{i,t-1} + X_{i,t} = O_{it} + L_{it}, \forall i,t, \tag{2}$$

The corresponding material flow is depicted in Figure 2.

Recall that demand for end items occur at time points $t \in T_e$. Suppose the demand profile for end items is given as $\{d_{it,} i \in I_e, t \in T_e\}$, then in order to make sure that the demand for all end items met, it is sufficient to fix the corresponding transfer batch sizes as:

$$L_{it} = d_{it,} i \in I_e, t \in T_e, \tag{3}$$

There are resource constraints restricting the amount that can be processed in a period,

$$\sum_i \rho_{ki} X_{it} \le r_k [\mathrm{T}_{t-1}, \mathrm{T}_t], \forall k, t. \tag{4}$$

The transfers can take place only if they are indicated to do so. This can be handled by a set of conditional constraints:

$$(Y_{it} = 1) \rightarrow (L_{it} \ge 0) \tag{5}$$
$$(Y_{it} = 0) \rightarrow (L_{it} = 0) \tag{6}$$

Transfer times are assumed to be negligible. If these times are significant, then the inter-stage transfers can be assumed as a separate "stage" in processing with appropriate resource requirements. In order to make sure that there is a material handling equipment available when required,

$$\sum_i Y_{it} \le \gamma, \forall t, \tag{7}$$

where γ is the number of material handling equipment available. If it takes a maximum of ε units of time for an interstage transfer to take place, then to ensure the availability of handling equipment at each period,

$$\mathrm{T}_t \ge \mathrm{T}_{t-1} + \varepsilon, \forall t, \tag{8}$$

and with the appropriate nonnegativity, integrality, and the initial conditions.

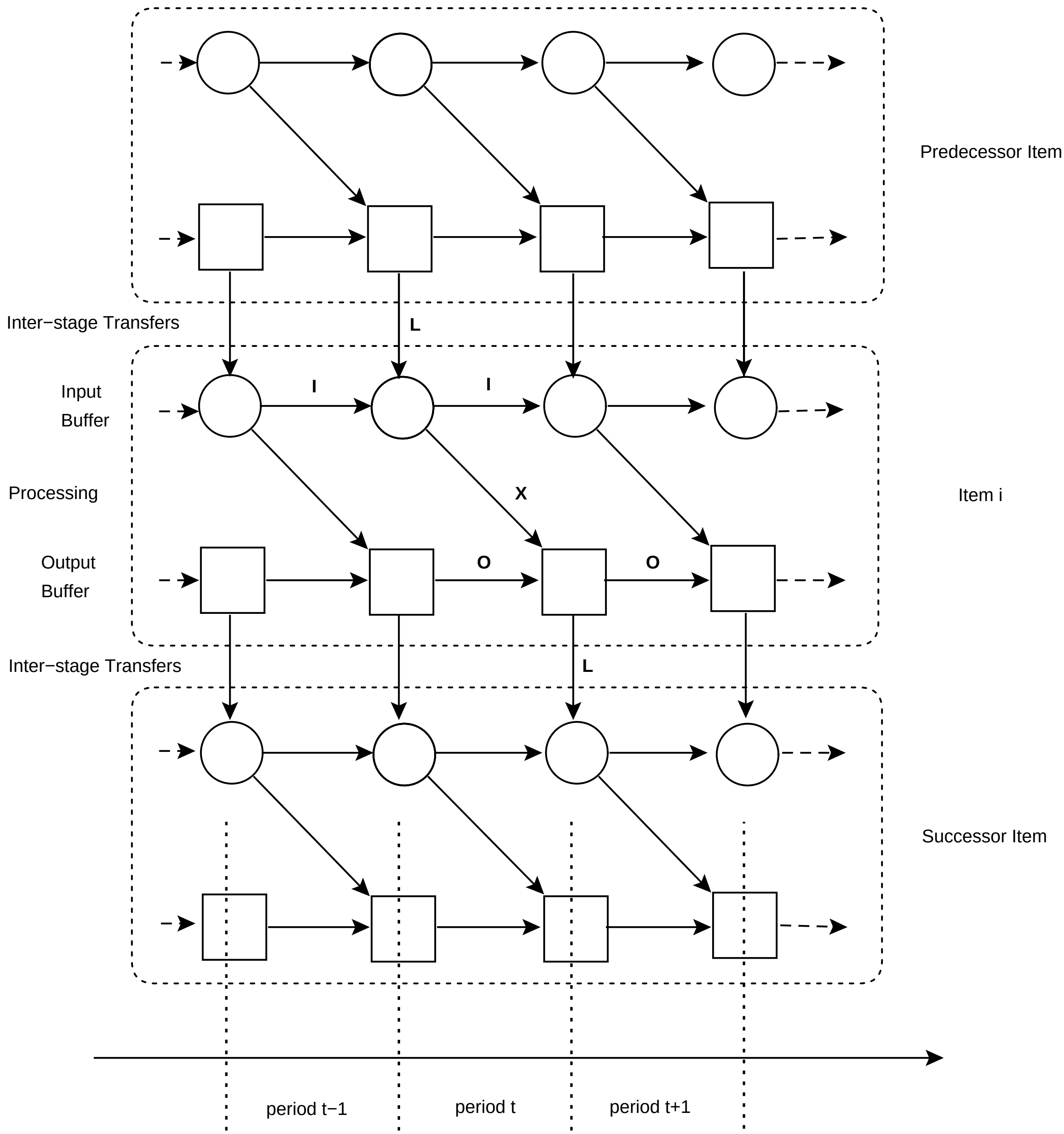

Figure 2. Material Flow

The essence of this model is how the exogenous events whose times are given and the endogenous events whose times of occurrence are decision variables are handled in the same model. How logic-based formulation handles this problem will be illustrated using a simple example.

Suppose there are two orders of different sizes of the same job. The deadlines for these "sublots" are τ_1 and τ_2, respectively. For this example, the set of items is $i \in I = \{1, \cdots, m\}$, whose index set of end items is a singleton, $I_e = \{m\}$; and predecessor and successor of item *i* are given by

$$\pi(i) = i - 1, i = 1, \cdots, m,$$
$$\sigma(i) = i + 1, i = 1, \cdots, m - 1. \qquad (9)$$

In addition to initializing the beginning inventory levels, let $\mathrm{T}_0 = 0, \mathrm{T}_\mathrm{n} = \tau_2$, and $L_{mn} = d_{\tau_2}$. That is, our planning horizon is from time 0 to the deadline of the second order, τ_2, at which time $L_{mn} = d_{\tau_2}$ units will have to be delivered. The problem is to determine the timing of interstage material transfers; one of which will be the deadline of the first order, τ_1. Assigning a time point T_t to the deadline of the first order, τ_1, can be handled by means of a global constraint. First we need to define the following set of functions:

$$f_j(\cdot) = O_{i,j-1} + X_{mj} - O_{mj}, \; j = 1, \cdots, n-1. \qquad (10)$$

Then the mixed extended element constraint,

$$\texttt{m-xelement}(y, (f_j(\cdot), \cdots, f_{n-1}(\cdot)), z), \qquad (11)$$

which is equivalent to the conditional constraint

$$(y = j) \rightarrow (z = f_j(\cdot)), \; j = 1, \cdots, n-1, \qquad (12)$$

can be used with the constraint $z = d_\tau$.

Conclusions

In this paper an event-time model for a scheduling production problem is presented. It is based on lot streaming paradigm and makes use of the basic modeling framework of constraint programming. The major significance of this modeling approach is that it consolidates the usable aspects of periodic and continuous review modeling paradigms of production planning. The computational advantages provided by constraint programming and the availability of user-friendly software makes this approach computationally viable.

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