Event-Time Models in Inventory Systems

Ömer S. Benli
California State University, Long Beach

INFORMS - Seattle
November 4-7, 2007

Motivation

Maxwell (1997)

criticized of the use of models that lump operations and events into large time buckets, and

suggested formulation of *"event-time"* models for production planning and scheduling.

Karmarkar (1994) argued that

in practice, it is much easier to think in terms of time lines and events and intervals. In some models, such as scheduling with Gantt charts, we use this kind of modeling, but we lack ways of dealing with decomposition or composition of these models.

As a result, capacity and planning models are often formulated very differently. Perhaps this is one reason that time interval and release oriented methods like MRP and DRP are used in practice for planning even though they are completely unable to actually deal with resource allocation decisions.

This work aims to provide computationally viable approaches

- to integrate production planning and scheduling using lot streaming paradigm *(Benli, 2007),*
- to modeling of multi-echelon inventory systems for supply chains *(Benli, under preparation).*

In this presentation, the concept of *Event-Time Models* is introduced by means of "a single-item capacitated production – inventory process."

Inventory Systems

- *Inventory* exists as a buffer between demand and procurement.
- Primary function of an *inventory system* is to decide how to meet this demand.
- This is achieved by *manipulating the timing and the size* of production or purchase orders.

Modeling Paradigms

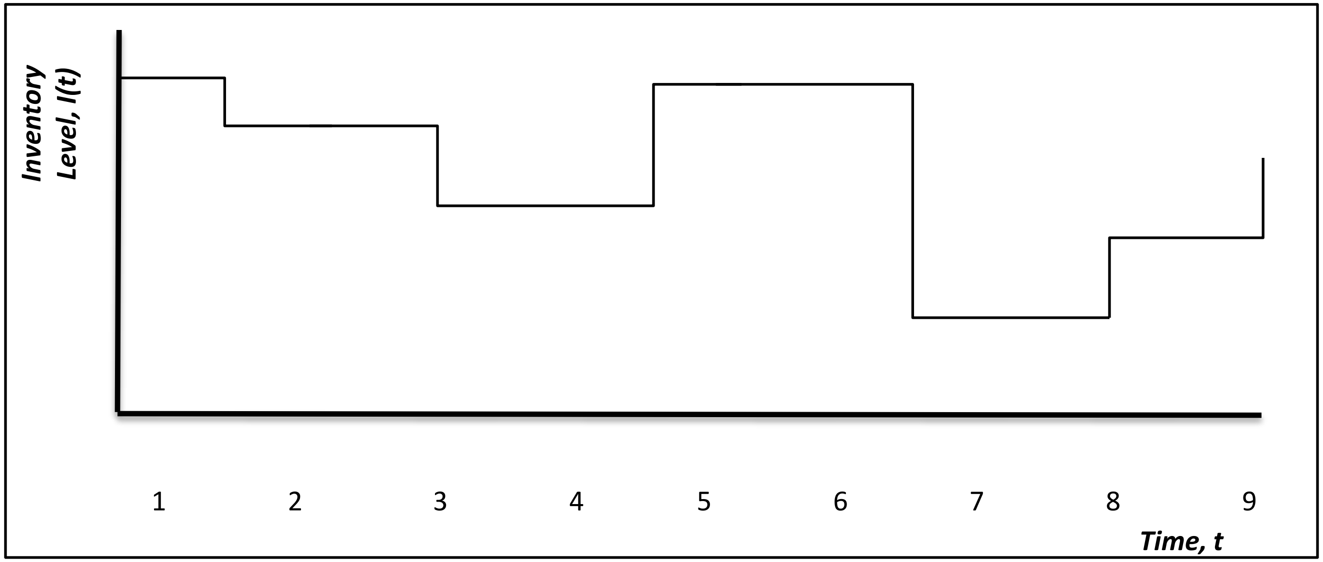
- Continuous Review Process

$$\mathbb{T} = \{\, 0 \leq t \leq T \,\}$$

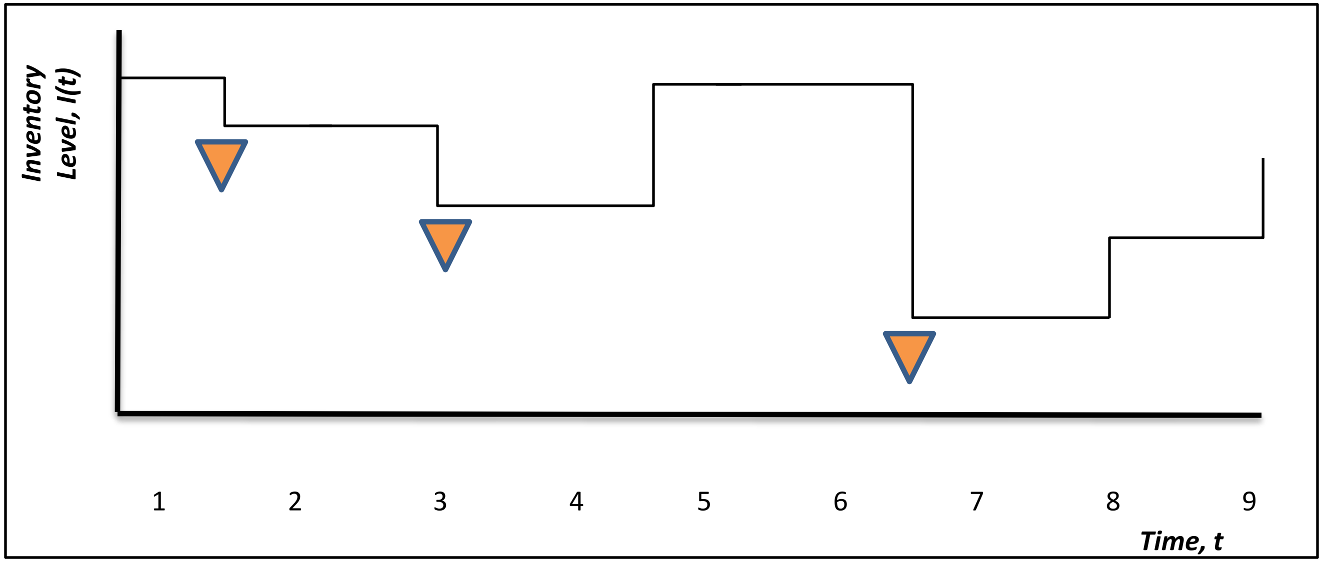
- Periodic Review Process

$$\mathbb{T} = \{\, t_0, t_1, \ldots, t_T \,\}$$

Continuous Review Process

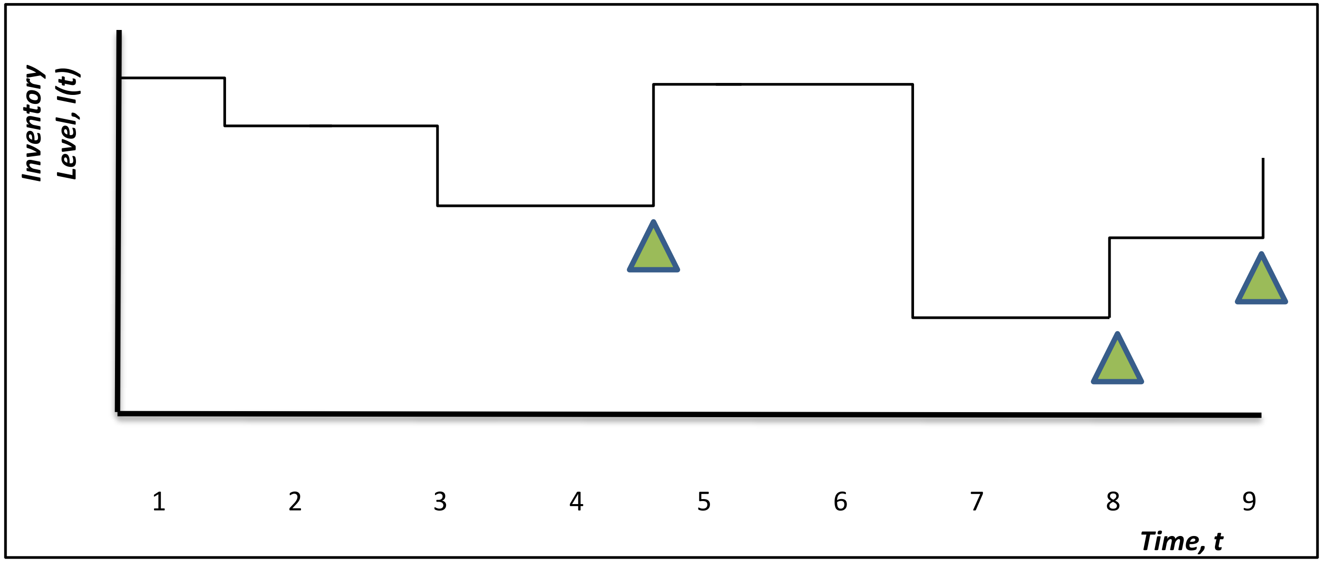


Continuous Review Process

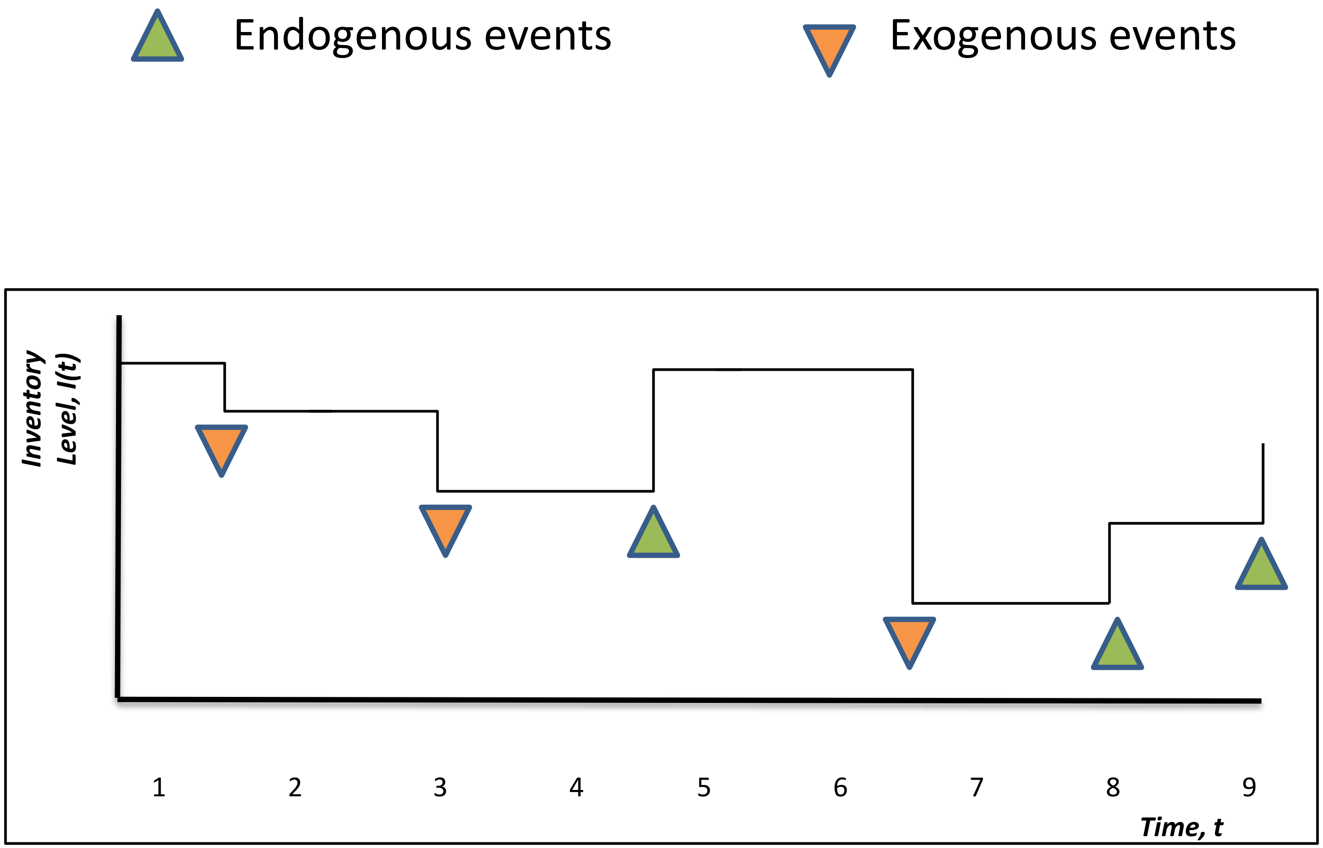


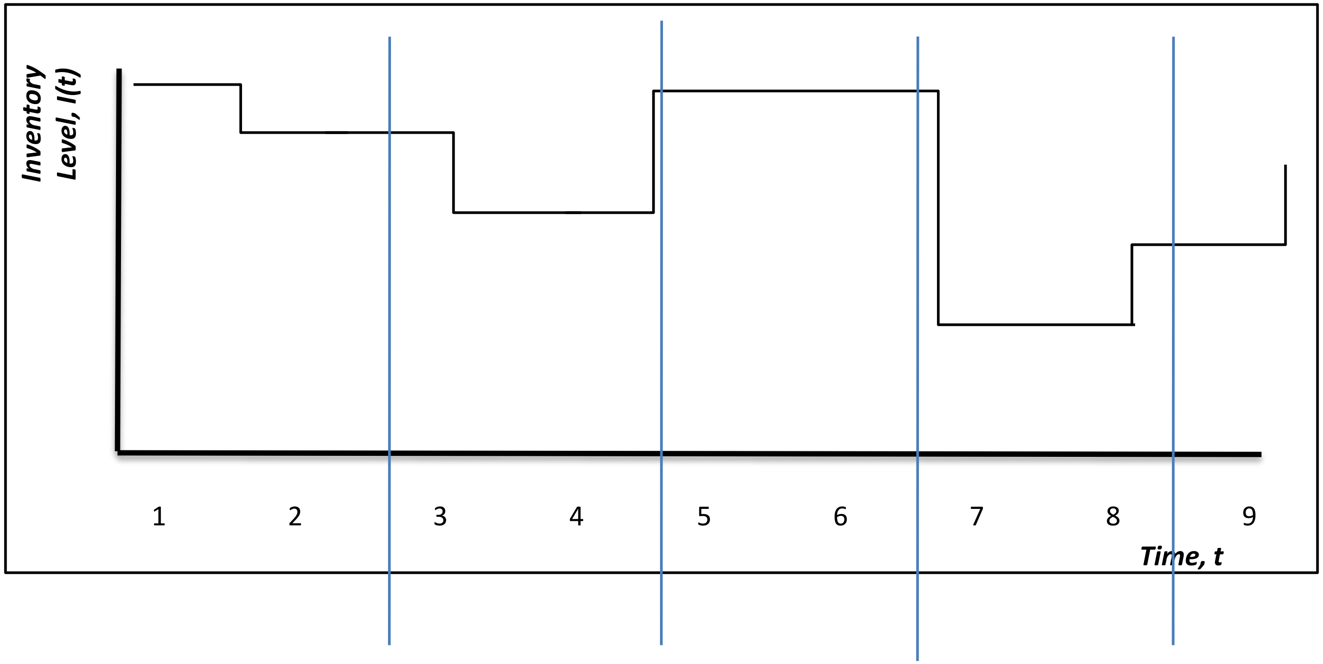
Continuous Review Process

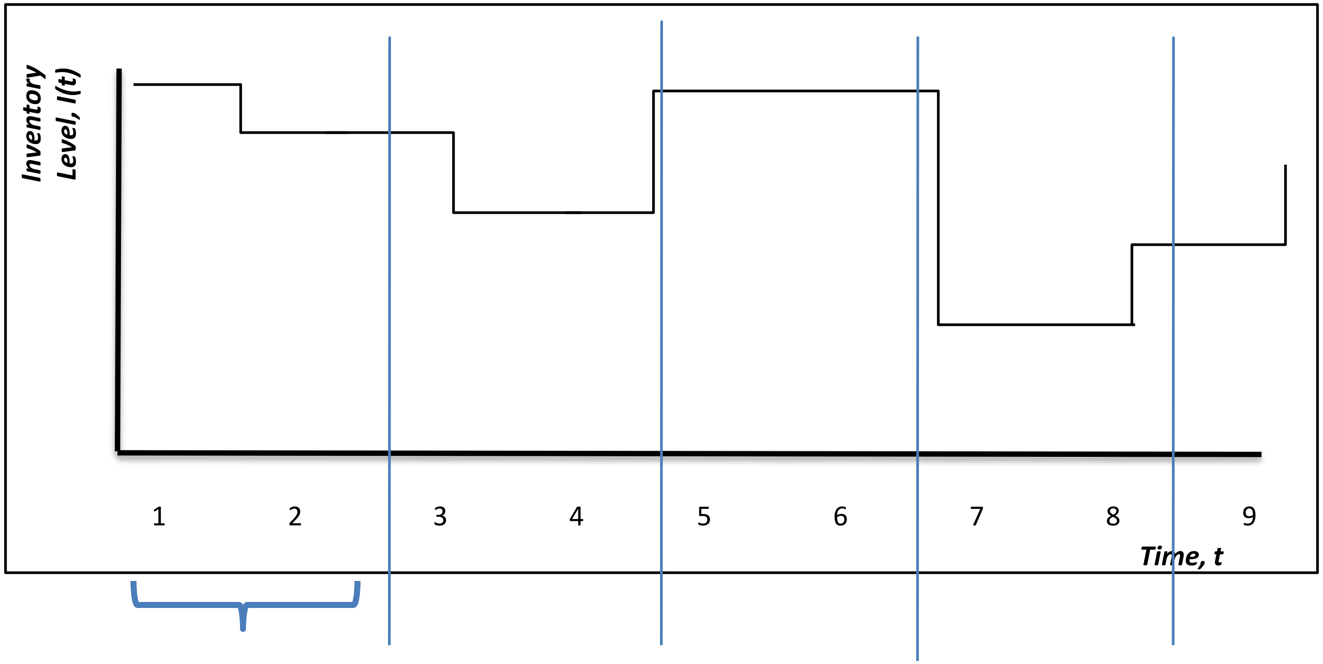
Endogenous events (procurement)

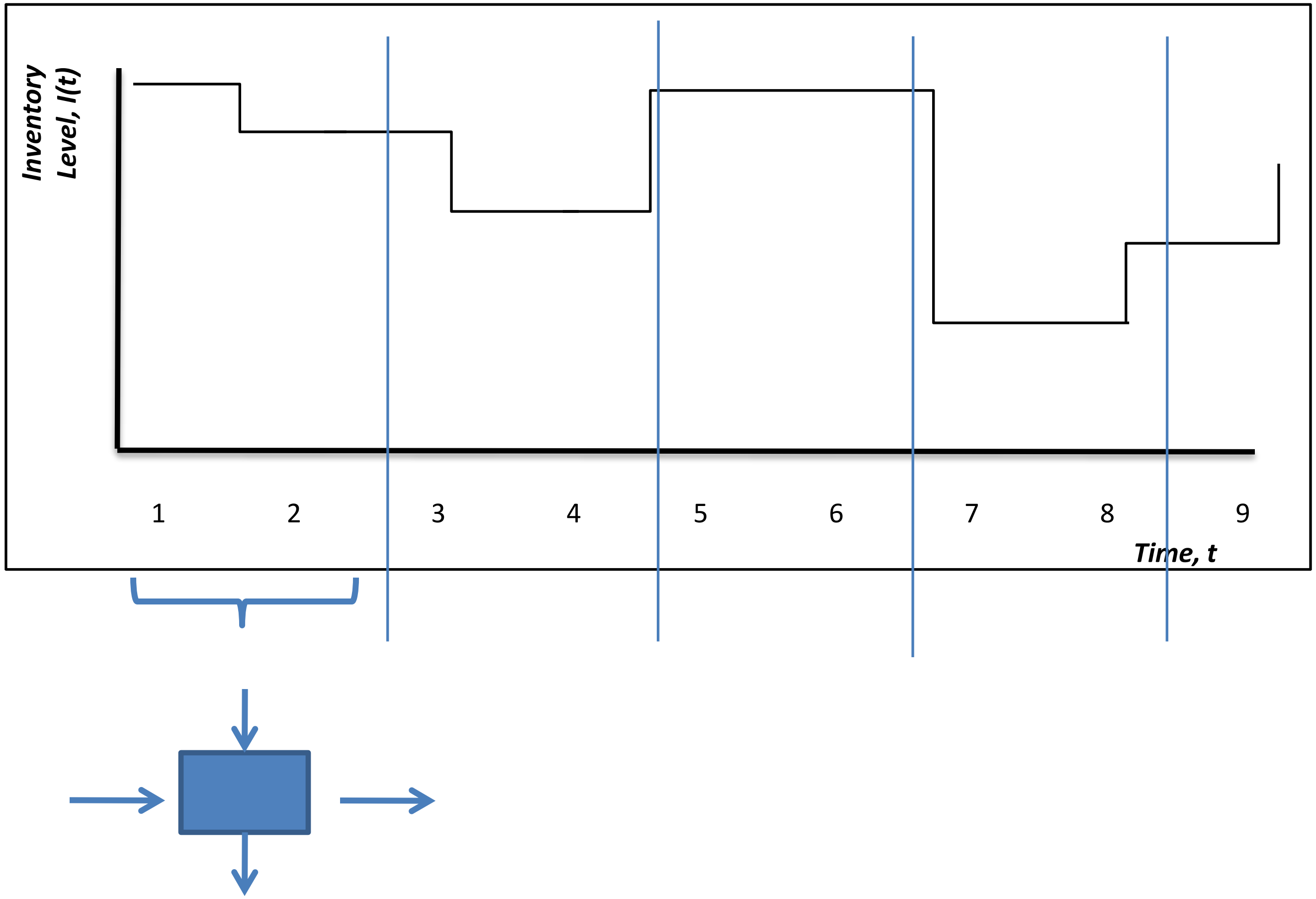


Continuous Review Process

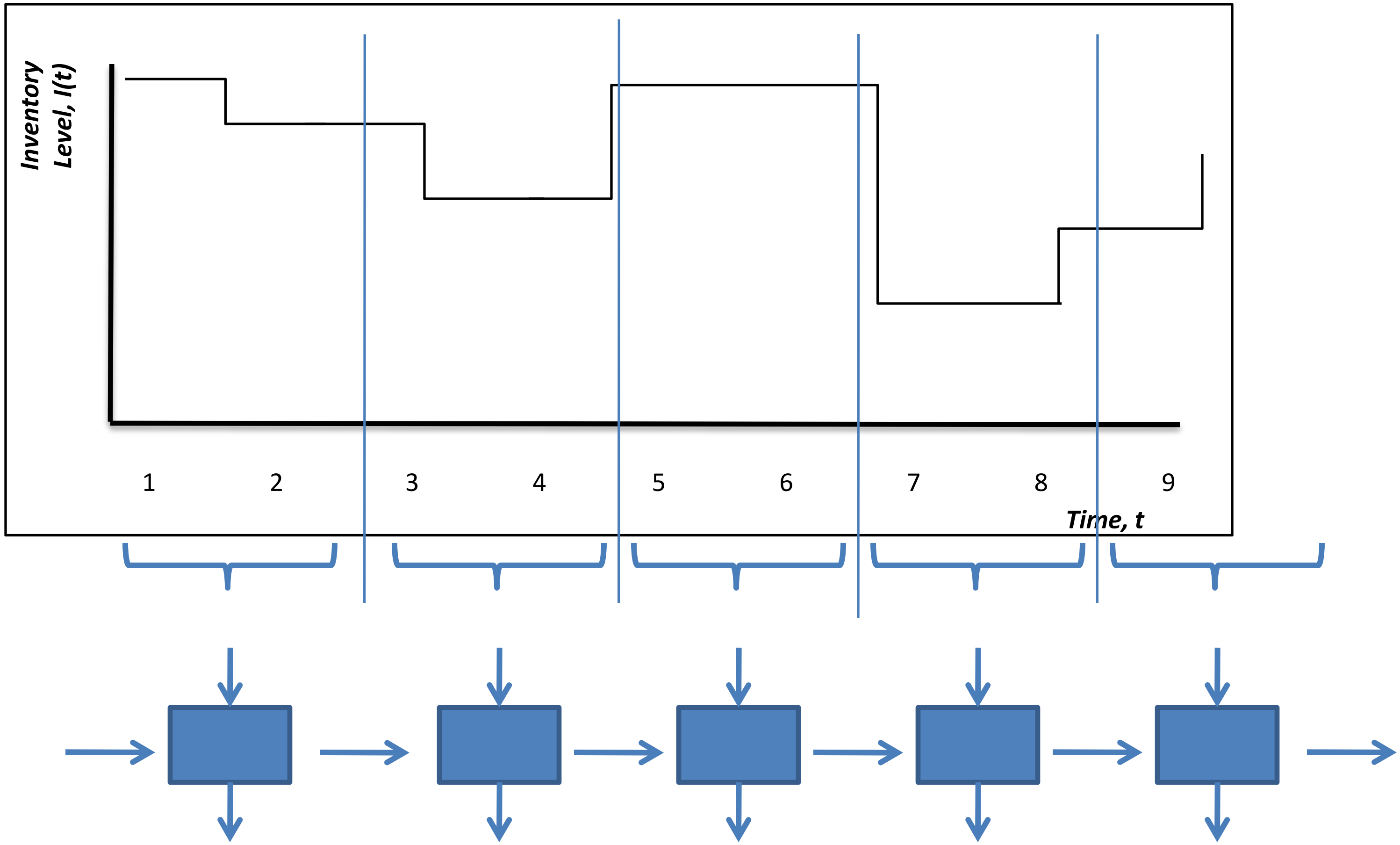



Inventory Level, I(t)
1
2
3
4
5
6
7
8
9
Time, t


Inventory Level, I(t)
1
2
3
4
5
6
7
8
9
Time, t

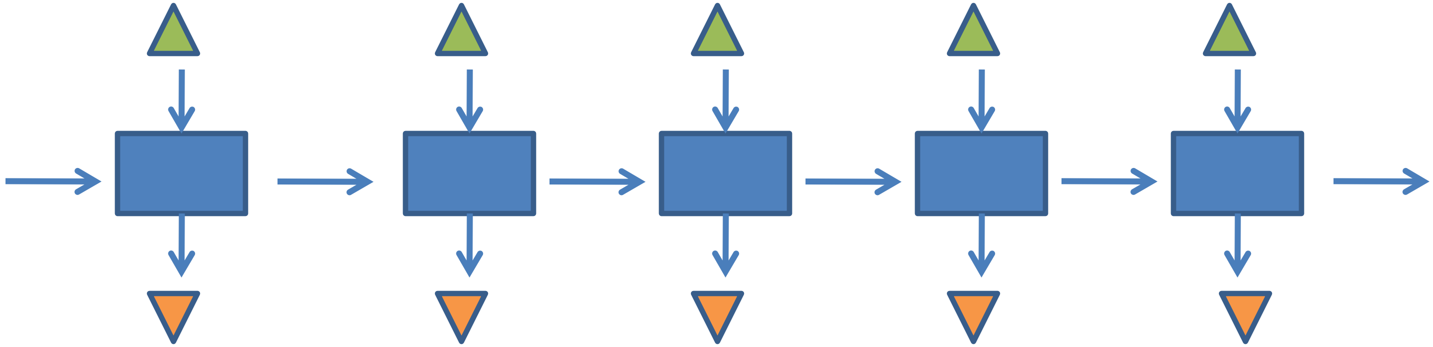


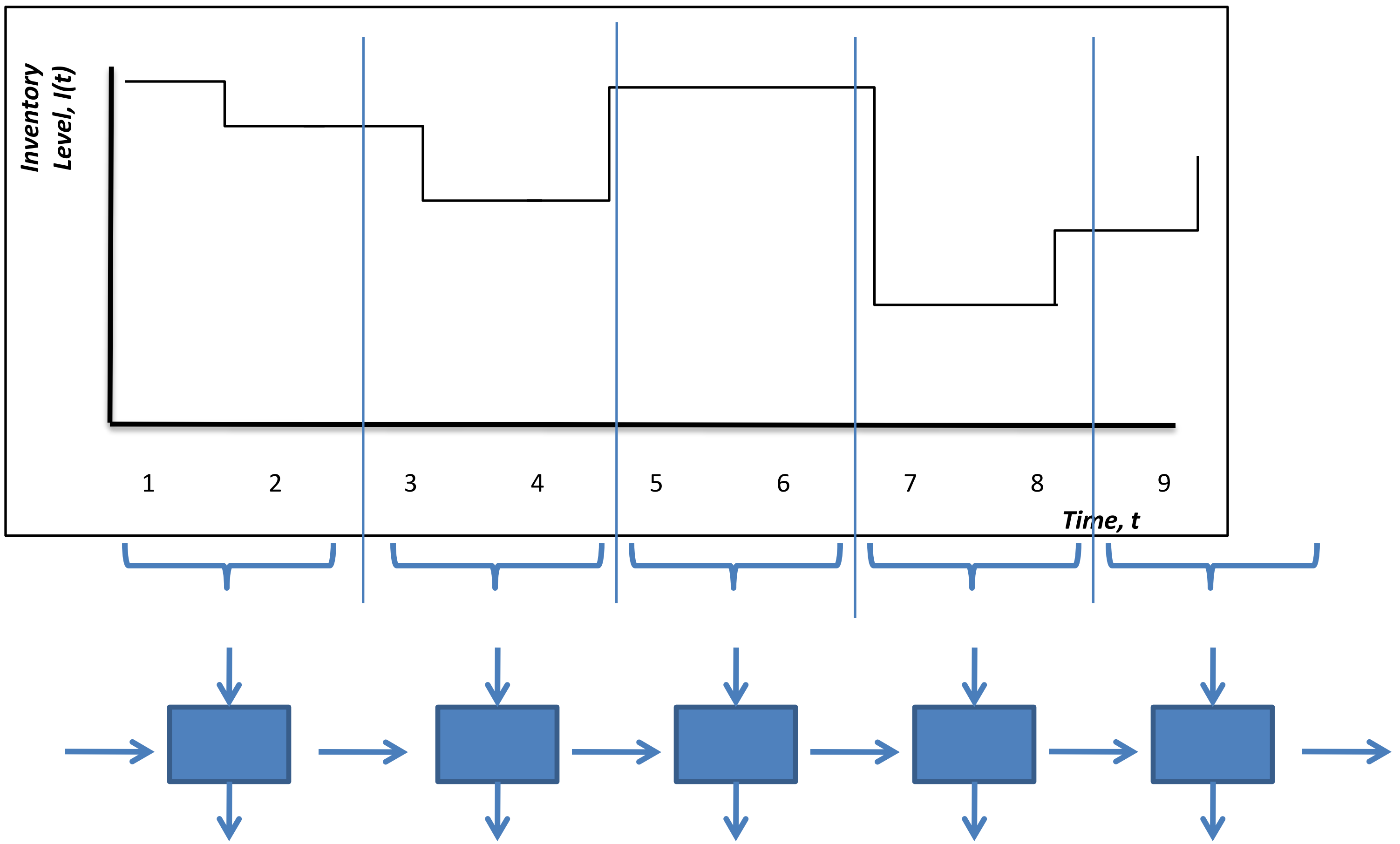
Inventory Level, I(t)
1
2
3
4
5
6
7
8
9
Time, t

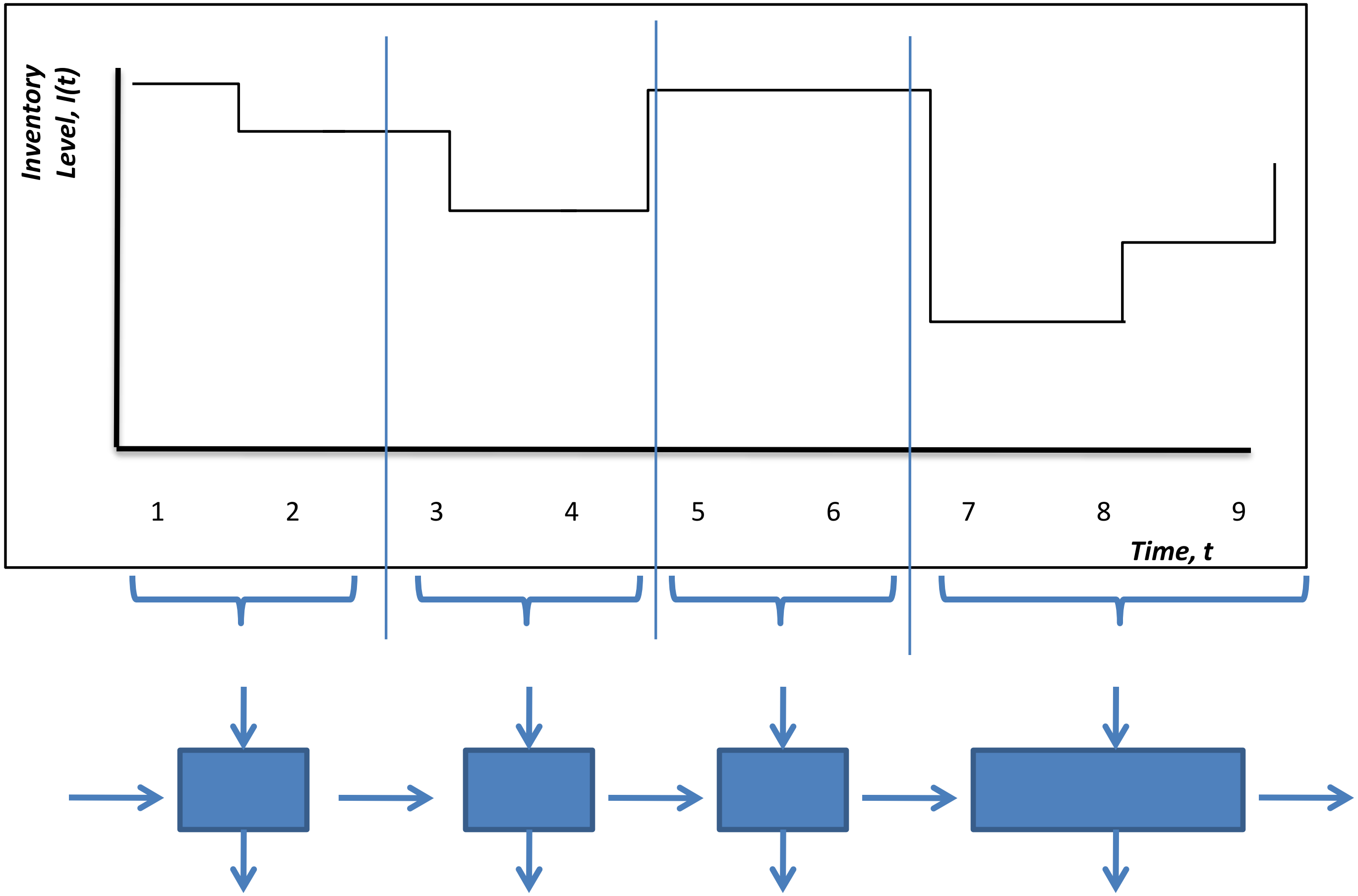

Inventory Level, I(t)
1
2
3
4
5
6
7
8
9
Time, t

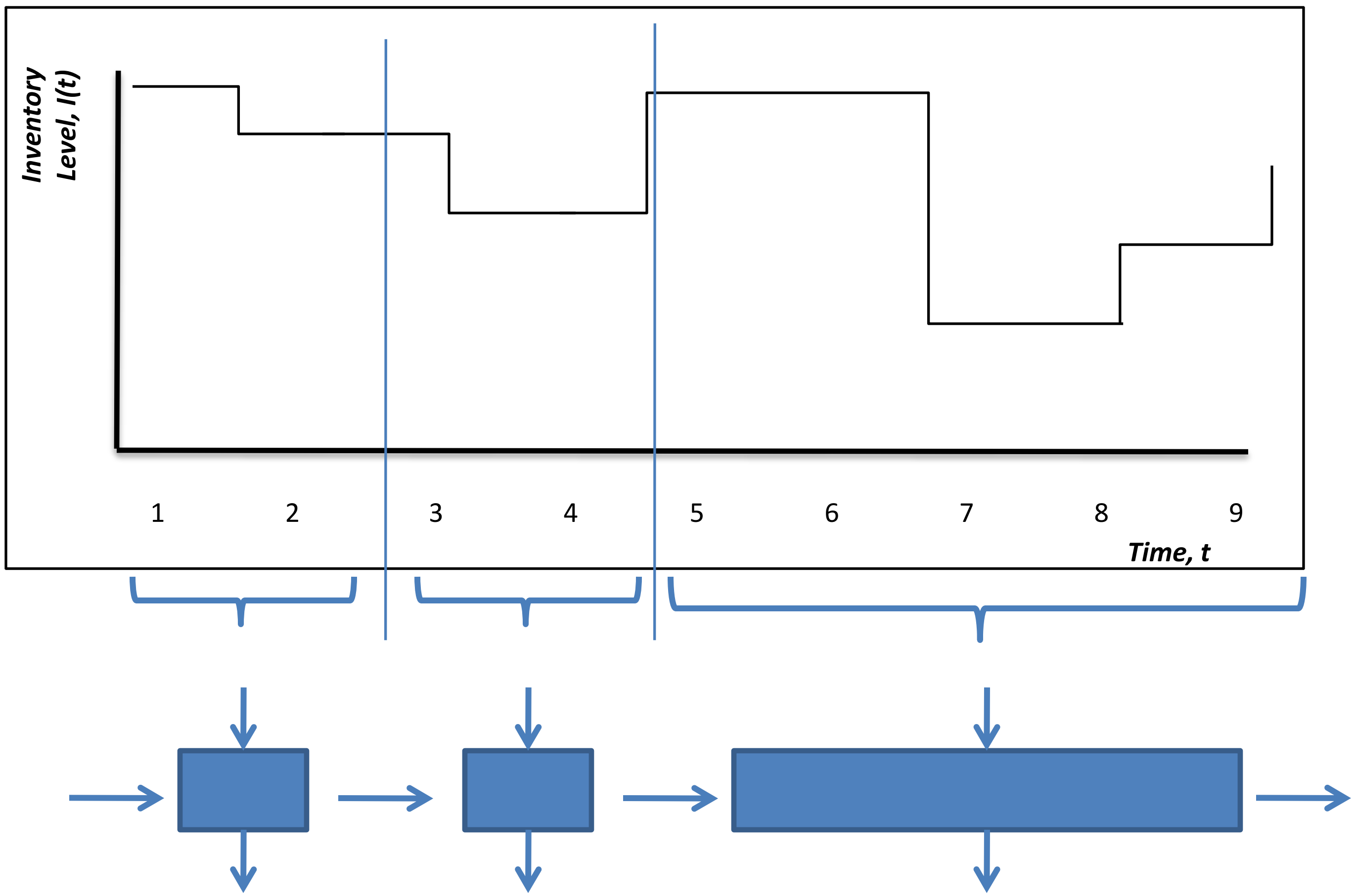
Periodic Review Process

▲ Endogenous events　　▼ Exogenous events




Inventory Level, I(t)
1
2
3
4
5
6
7
8
9
Time, t

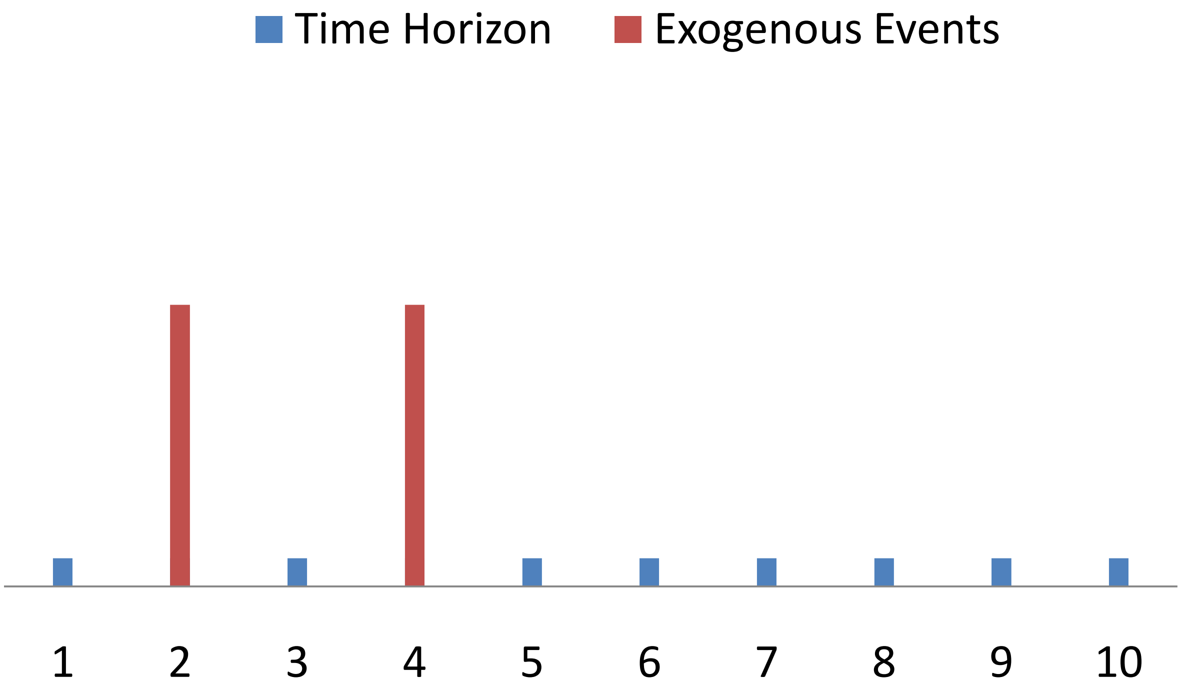

Inventory Level, I(t)
1
2
3
4
5
6
7
8
9
Time, t


Inventory Level, I(t)
1
2
3
4
5
6
7
8
9
Time, t

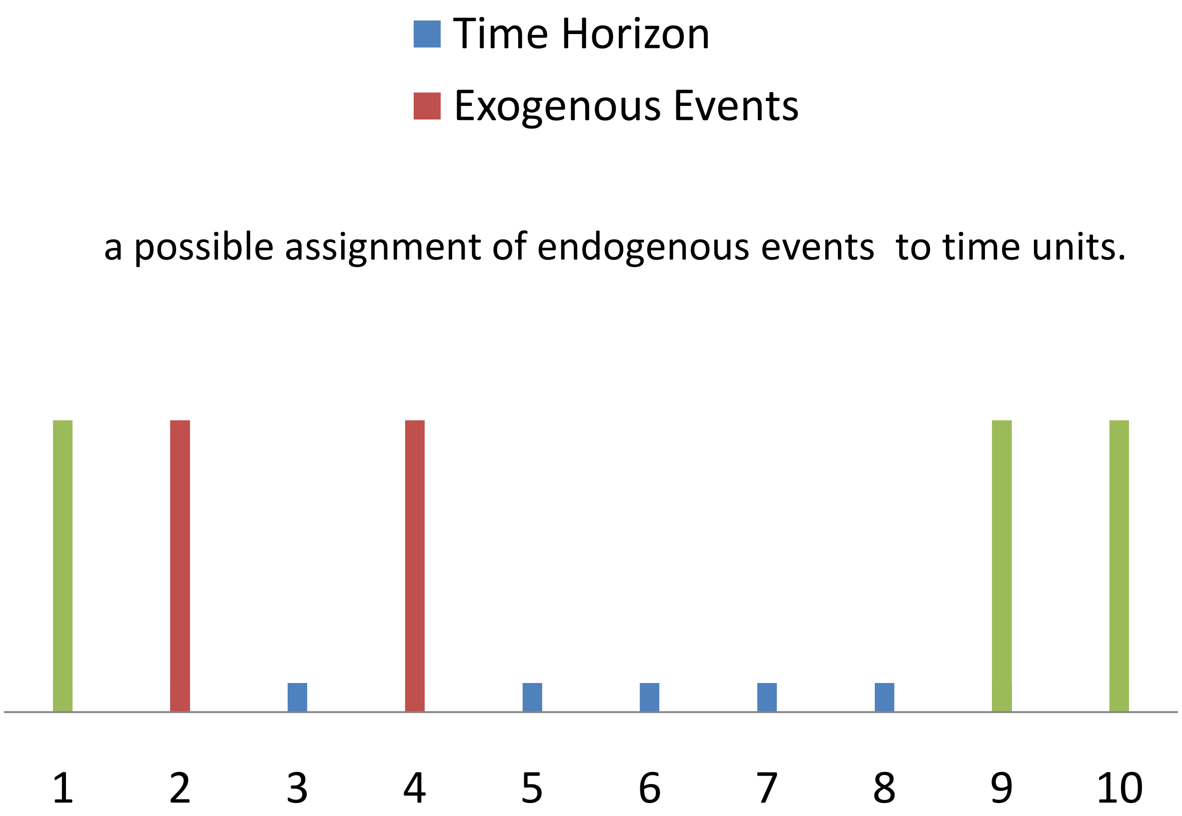
How to partition the planning horizon into periods?

- Consider
  - a planning horizon of 10 units of time,
  - two demand occurrences (exogenous events) at
    - time unit 2, and
    - time unit 4.
- Assume
  - a maximum of three planned procurements (endogenous events)

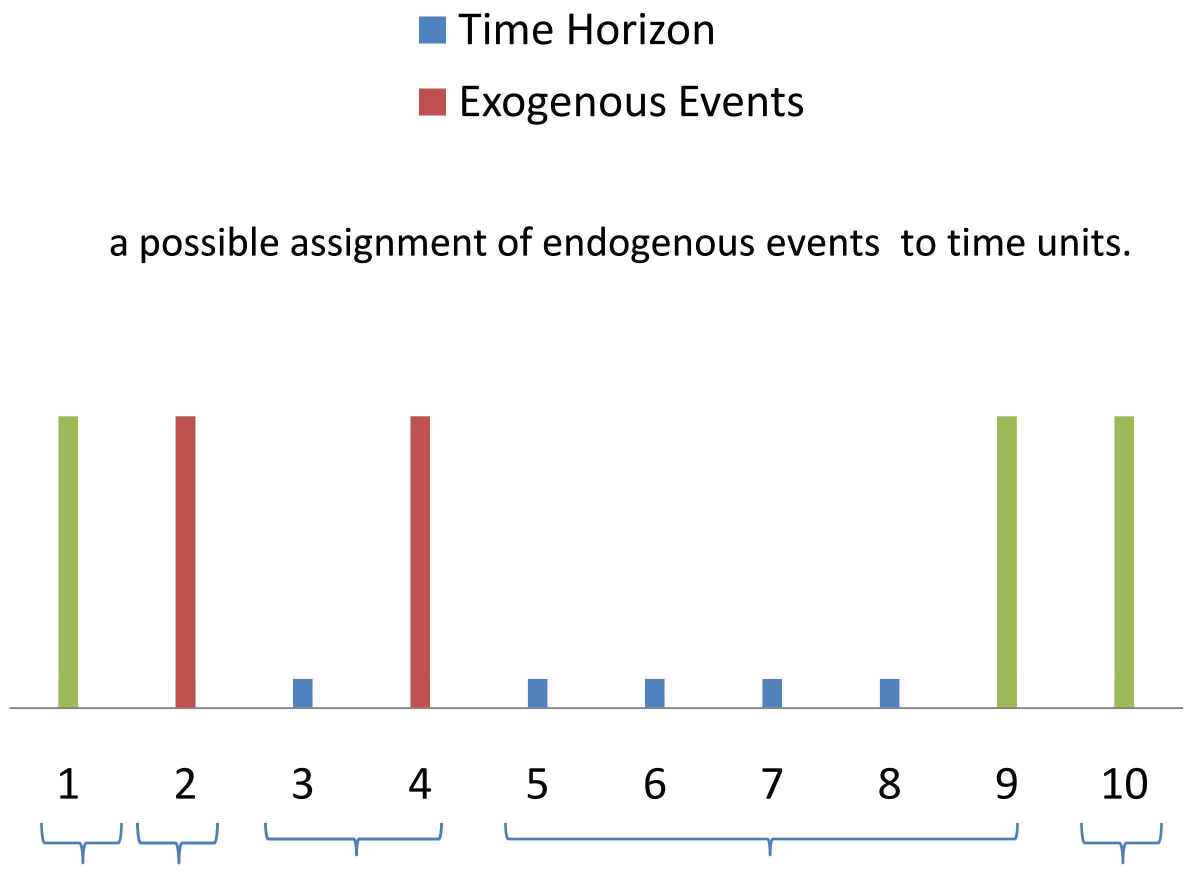
- Consider
  - a planning horizon of 10 units of time,
  - two demand occurrences (exogenous events) at
    - time unit 2, and
    - time unit 4.
- Assume
  - a maximum of three planned procurements (endogenous events)



- Consider
  - a planning horizon of 10 units of time,
  - two demand occurrences (exogenous events) at
    - time unit 2, and
    - time unit 4.
- Assume
  - a maximum of three planned procurements (endogenous events)



- Consider
  - a planning horizon of 10 units of time,
  - two demand occurrences (exogenous events) at
    - time unit 2, and
    - time unit 4.
- Assume
  - a maximum of three planned procurements (endogenous events)



Event-Time Models

A “period” t, with a variable length, is defined as the time interval

$$[T_{t-1}, T_t], t \in \mathbb{T} = \{1, \ldots, n\}.$$

These time points are decision variables of the problem, except for the points corresponding to exogenous events (demand occurrences.)

Let a subset of

$$\mathbb{T} \supset \mathbb{T}_{ex} = \{\tau_1, \tau_2, \ldots, \tau_m\}$$

denote the time points corresponding to exogenous events.

An event-time model for a single-item capacitated production – inventory process

Let

I_t : inventory level at the end of period *t*, T_t .

X_t : procurement during period *t*, $[T_{t-1}, T_t]$,

D_t : demand at time T_t if $t \in \mathcal{T}_{ex}$ *,otherwise it is equal to zero.*

$$I_{t-1} + X_t = I_t + D_t \text{, all } t \in \mathcal{T}$$

$$X_t \leq \rho\, [T_t - T_{t-1}] \text{, all } t \in \mathcal{T}$$

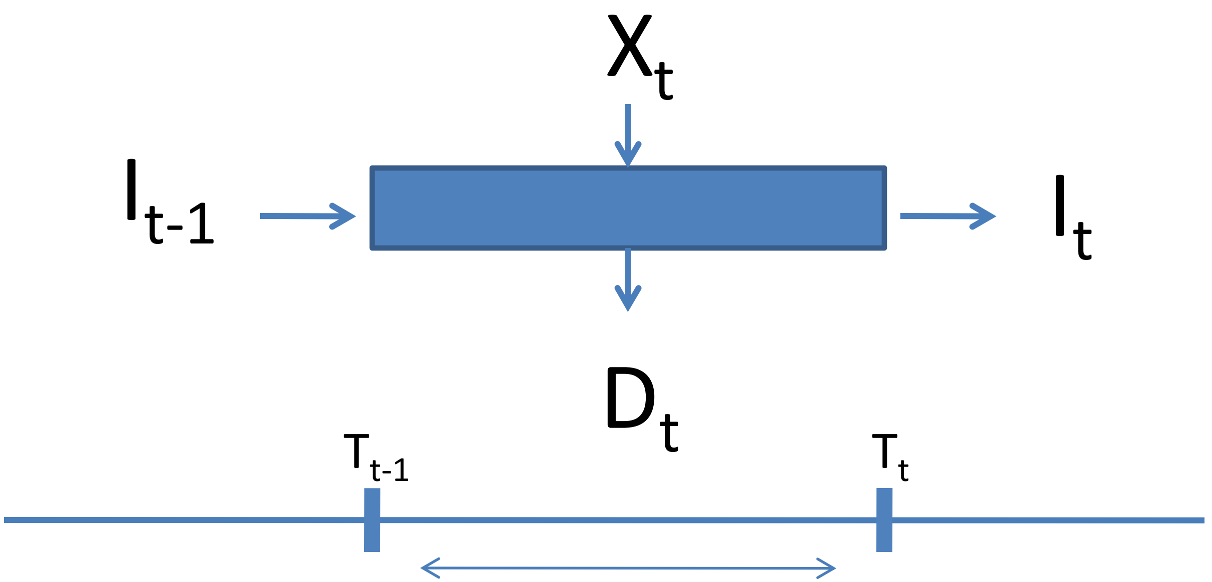
where ρ = procurement rate.

An event-time model for a single-item capacitated production – inventory process

$$I_{t-1} + X_t = I_t + D_t, \text{ all } t \in \mathcal{T}$$

$$X_t \le \rho\,[T_t - T_{t-1}], \text{ all } t \in \mathcal{T}$$

where ρ = procurement rate.



How to fix the given times of exogenous events at specific periods?

- Let ν_k denote the period at the end of which an exogenous event, *k*, occurs.
- $\mathbb{T}_{ex} = \{\tau_1, \tau_2, . . . , \tau_m\}$ are the times at which the exogenous events occur,
- Consider the following <u>*conditional constraint*</u> that makes use of <u>*variable subscripts*</u>:

$$(\nu_k = t) \rightarrow (T_t = \tau_k),$$

$t \in \mathbb{T} = \{1, . . . , n\},$

$\tau \in \mathbb{T}_{ex} = \{\tau_1, \tau_2, . . . , \tau_m\}$, and

for each $k = 1, . . ., m.$

Portion of the OPL Code

```
int nbPeriods = 5;
int nbEvents = 2;
int horizon = 10;

range period 1..nbPeriods;
range exoEvent 1..nbEvents;

int givenTime[exoEvent] = [3,8];

var int time[period] in 1..horizon;
var int index[exoEvent] in 1..nbPeriods;

solve{
forall(t in 2..nbPeriods)
time[t-1] < time[t];

forall(i in exoEvent)
time[index[i]] = givenTime[i];
}
;
```

Portion of the OPL Code

```
int nbPeriods = 5;
int nbEvents = 2;
int horizon = 10;

range period 1..nbPeriods;
range exoEvent 1..nbEvents;

int givenTime[exoEvent] = [3,8];

var int time[period] in 1..horizon;
var int index[exoEvent] in 1..nbPeriods;

solve{
forall(t in 2..nbPeriods)
time[t-1] < time[t];

forall(i in exoEvent)
time[index[i]] = givenTime[i];
}
;
```

Total number of periods in the model: *n*

Portion of the OPL Code

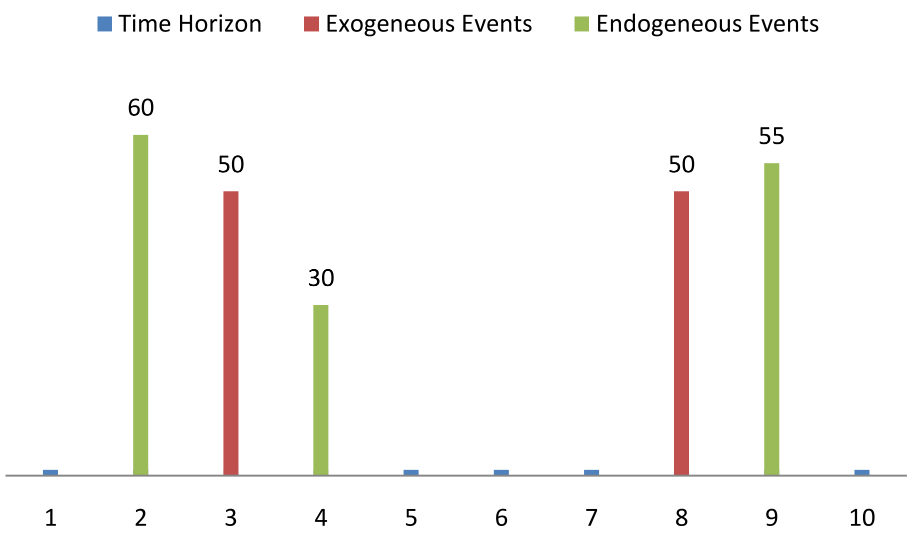
```
int nbPeriods = 5;
int nbEvents = 2;
int horizon = 10;

range period 1..nbPeriods;
range exoEvent 1..nbEvents;

int givenTime[exoEvent] = [3,8];

var int time[period] in 1..horizon;
var int index[exoEvent] in 1..nbPeriods;

solve{
forall(t in 2..nbPeriods)
time[t-1] < time[t];

forall(i in exoEvent)
time[index[i]] = givenTime[i];
}
;
```

Total number of exogenous events: m

Portion of the OPL Code

```
int nbPeriods = 5;
int nbEvents = 2;
int horizon = 10;

range period 1..nbPeriods;
range exoEvent 1..nbEvents;

int givenTime[exoEvent] = [3,8];

var int time[period] in 1..horizon;
var int index[exoEvent] in 1..nbPeriods;

solve{
forall(t in 2..nbPeriods)
time[t-1] < time[t];

forall(i in exoEvent)
time[index[i]] = givenTime[i];
}
;
```

The times of exogenous events $\mathbb{T}_{ex} = \{\tau_1 \;, \tau_2\}$



Time Horizon
Exogeneous Events
Endogeneous Events
60
50
30
50
55
1
2
3
4
5
6
7
8
9
10

Event-Time Models

- provide a convenient way to integrate continuous review (“scheduling”) and periodic review (“production planning”) models of inventory systems,
- provide a general scheme for modeling multi-echelon inventory systems for supply chains.