## TAKE-HOME EXP \#5A The Location, Velocity, and Acceleration of a Bouncing Ball

First, we'll do some exercises that will give you a "feel for" the phenomena and representations of their relationships in graphs.

Then we will examine the tricky motion of a bouncing ball. Actually it's not the motion that's tricky, it's how you communicate it. The real "trick" is to connect what you see with graphic visualizations of the location, velocity, and acceleration of the ball as time passes.

A racquetball or tennis ball can do this job, but a "super" elastic ball works best because it bounces higher for a longer time. Drop the ball from about 1.5 meters. What do you observe? I'm interested in looking at the time evolution of the vertical location of the ball, of its velocity (magnitude and direction), and of its acceleration. Three graphs can describe the motion of the bouncing ball:

1) location vs. time,
2) velocity vs. time, and
3) acceleration vs. time.

The graphs themselves will be qualitative, and capture the general features of the motion. That means that only estimates of relative effects will be important. We will assume that air friction plays no significant role in changing the motion, and this is a very good assumption at these low speeds. I'll start you off with the first graph, and give some hints about the others.

## Part I. H ow can one get a good operational sense of the motion quantities? <br> (adapted from A. Arons, A Guide . . ., J ohn Wiley and Sons, NY, 1990)

It's not easy to separate the concepts of location and displacement and the latter's change as time passes, producing acceleration and velocity. The exercises below are easy to do, alone or with friends to check your movements. They can give you a body-sense experience to relate to these ideas.

1. First, let's set up a straight-line track, a one-dimensional line along which you can freely move. Let the straight edge of the long side of a table or countertop be the straight-line track. Use a pen, book-edge, or anything to mark the center of this "track" as the zero reference. Assume locations to the right are positive, that is a displacement from zero to the
 right will be our choice for a " + " direction. Locations to the left will be negative ("-"). Your own hand will be the moving object.
a) Interpret each diagram below by performing the indicated motion with your hand.

Describe the motion in words as you execute it. Include all the descriptive details, such as speeding up. slowing down, reversing direction, standing still as time passes, moving with constant velocity, having your hand at the appropriate location at time $t=$ 0 (the value assigned to whatever clock-reading with which you start), and so on.

Note the labels on the axes of the following graphs. The graphs give histories (events as time passes) of location versus clock-readings. The axis are marked very generally, with no particular units specified.




Advice: As I, the friendly physicist, did the exercises, I found that it was useful to take 5 to 10 seconds, at least, to complete the motion depicted by the graph. The graph is a history with time marching along the horizontal axis. Since you are working along the edge of a table, the distance units can be assumed to be about the size of centimeters or inches $(1 \mathrm{~cm}$ is slightly under $1 / 2$-inch; $1 \mathrm{inch} \approx 2.5 \mathrm{~cm}$ ).

Procedure: First locate time $t=0$ at the place where the axes cross. Place your hand at the location that corresponds to $t=0$. Your eye moves an increment along the horizontal axis, and then scan vertically up or down to find the location of the hand, and place it there. At first, this will result in a choppy, rather than continuous motion of the hand. But once you do a particular history slowly, you can probably repeat it more smoothly. Ask a partner or someone whether they agree that your motion is the one depicted.
b) The graphs below give histories of velocity versus clock-readings.

The axis are again marked very generally, with no particular units specified.
Assume units consistent with the advice given above.



c) After you have done the motions and described them in words, then for Graphs A, B, and C, sketch below each of them the velocity versus time graph that is appropriate for each. Note that the velocity axes have plus (rightward) and negative (leftward) markings, since velocity is a vector. Graphs $\mathrm{A}, \mathrm{B}$, and C are redrawn below, and, below each of them, v vs. $t$ axes for your use.






2. a) Use the same straight-line table-edge as a one-dimensional track. This time the task is to interpret the following histories depicted by the velocity versus clock-
reading graphs. Once again, describe the motion in words as you execute it.
ALERT: Since the graphs do not specify locations, they don't tell you where the graphed motion starts, that is, the location of your hand when the clock starts at $t=0$. To see if this makes a difference, execute each motion more than once, each time placing your hand at a different initial position at $\mathrm{t}=0$.






b) After you have done the motions and described them in words, then for Graphs $A 1, A 2$, and $B$, then sketch above each the graph of location versus clock-reading that is appropriate for each. Graphs A1, A2, and B are redrawn below, and, above them, $x$ vs. $t$ axes for your use.


NAME $\qquad$ ID\# $\qquad$
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Part I D escription Sheet: Please answer the following about the graphs on the previous pages:
1a. Identify the kind of graph, shown to the right. Describe the motion in words as you execute it. Include all the descriptive details, such as speeding up. slowing down, reversing direction, standing still as time passes, moving with constant velocity, having your hand at the appropriate location at time $t=0$ (the value assigned to whatever clock-reading with which you start), and so on. (Use other side, if needed.)

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1b. Identify the kind of graph, shown to the right. Describe the motion in words as you execute it. Include all the descriptive details, such as speeding up. slowing down, reversing direction, standing still as time passes, moving with constant velocity, having your hand at the appropriate location at time $t=0$ (the value assigned to whatever clock-reading with which you start), and so on. (Use other side, if needed.)


## Part II. History of a Bouncing Ball

Work in teams of 3 or 4 people, if at all possible. At the end of the observations, each person in the group can make a velocity graph and explain them to the others. All you can do is make a mistake and learn something from it. There are no other "penalties". Take a chance when the time comes, and make your own graph so you can compare it to the others.

First, drop a ball from rest at a height of about 2 meters. The height depends on the bounciness of your ball; a "superball" can be dropped from a lower height.

1. During this experiment, drop the ball as many times as you wish from a measured height. As you observe the bounces, note where the velocity is zero in the vertical direction, and where it appears to have its maximum value. Although the ball might drift to the side one way or other, we are interested only in the vertical motion, the one affected by the acceleration due to gravity. Please agree with all your colleagues in the observation about where in the motion the vertical velocity is zero and where it has its largest value
2. When the ball is released from rest, its velocity is zero at that instant. It increases its speed on the way down solely due to gravitational acceleration, $\varnothing$ $=-9.8 \quad$. The negative sign comes from my implicit assignment of directions to the vertical number line, with "positive" meaning upward in the local environment, and "negative" meaning downward.

Let's make the directional assignment explicit in order to clarify the graphs:

- for location, the floor will be the reference zero, and any location above it will be positive. For example, if an initial drop height is 2 meters above the floor that could be recorded as " y 1 = + 2 meters". The label " y " for the vertical axis simply references the general labels for the axes, $x-y$ axes that are used in maps.
- for velocity and acceleration, the downward direction will be "negative" and the upward direction will be "positive". For example, with this choice of description, the gravitational acceleration will always be negative, $-9.8 \mathrm{~m} / \mathrm{s}^{2}$, where the negative sign carries the direction information.

Also, note that in the first instants after you release the ball and before it hits the floor, the velocity of the ball is negative, meaning it's heading downward vertically.
3. Many rubber balls bounce 3 or 4 times when dropped from 3 meters. Call the vertical direction the $y$-axis. Rather than spend the time making difficult height measurements, allow me to provide an example plot of the vertical height of a bouncing ball as time passes. Any information about the ball drifting horizontally sideways is not contained in this representation.


Precise times are on the graph in order to provide reference markers for the graphs that follow.
Note the following in this picture of the relationship between vertical location versus time:
a) The ball's height gets lower with each bounce.
b) After the moment of release, the ball hits the ground at times of about 0.8
 $\mathrm{s}, 2.1 \mathrm{~s}$, and 3.5 s .
4. It is also relatively easy to provide the acceleration vs. time graph, because, except the very short times the ball is in contact with the floor, the ball is in free fall subject only to gravity's $-9.8 \mathrm{~m} / \mathrm{s}^{2}$, with "downward" taken as the negative direction. That's the only acceleration there is-except for the moments of impact of the ball with the ground. Each impact provides the ball with a large positive upward impulse. It is only during the very brief times of contact with the floor, about $0.8 \mathrm{~s}, 2.1 \mathrm{~s}$, and 3.5 s after the drop, that the impact acceleration can occur. Therefore, the picture of the relationship of acceleration versus time is
 primarily a straight line with a zero slope.
5. Finally we come to the question. What does a picture of the relationship between the ball's velocity versus time look like?

On the bottom of the next page is a template for a $v_{y}$ vs. $t$ graph.
I have lined it up so the time axes line up with the previous graphical information. The times of impact with the ground are marked with vertical dotted lines that extend through all three graphs. Find those vertical lines now before proceeding, so you know how they connect the three graphs.

All that is wanted is a qualitative graph, roughly scaled. Make your best reasoned guess as to what the velocity looks like as a function of time passing, given your observation of the ball, and your knowledge of the location and acceleration information. What does the $v \mathrm{y}$ vs. time graph look like?

Hints: Since the ball starts from rest, your pencil must be initially at zero. Does the then move positively or negatively? One last hint for the friendly physicist's data to help you roughly scale your graph: at the moment just before the ball hits the ground for the very first time, its velocity will be $-8 \mathrm{~m} / \mathrm{s}$.


One thing you may be able to note from the powerful representation of your graph is the following. Although the ball does reach a zero value of the velocity at the highest place in its bounce, it only has that value for a single moment, a single clock-reading. The ball does not stop at the top, if by "stop" you mean have the value of zero velocity for a few instants. Your graph can show this fact clearly.

## Analysis Sheet: Please answer the following about the graphs on the previous page:

2a. At what times after the initial time does the ball reach its highest points above the ground? Locate those times on the graph, and write the times below.
$\qquad$ sec $\qquad$ sec $\qquad$ sec
$\mathbf{2 b}$. At what times during the bounces does the ball have zero speed?
$\qquad$ sec $\qquad$ sec $\qquad$ sec $\qquad$ sec $\qquad$ sec $\qquad$ sec

2c. The ball spends more time near its highest location than near any other location during a given rise and fall. From the graphical data, can you show this assertion to be true? Here's how:
a) On the location vs. t graph, draw some horizontal lines halfway from the maximum height of each peak, and obtain the amount of time the ball spends above this level compared to the total time of that particular bounce from impact to impact.
b) Describe what you find for these times, and state explicitly why the ball spends more time above the half-height level than below it.
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2d. The $y$-velocity vs. time graph should have slanting straight lines between contact times with the floor. (See the next page.) Why are those lines straight, and why do they slant downwards?
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