☐ KINETIC THEORY - CH. 17

WHERE DOES $PV = NkT$ COME FROM?

RELATES EACH THING A GAS CAN "KNOW" ABOUT ITSELF.

P - HOW MUCH IT IS BEING SQUEEZED

V - HOW BIG A SPACE DOES THE GAS LIVE IN

N - HOW MUCH GAS IS THERE

T - SOMETHING ABOUT HOW FAST THE GAS PARTICLES MOVE.

PRACTICAL: WHAT DOES THIS MEAN?

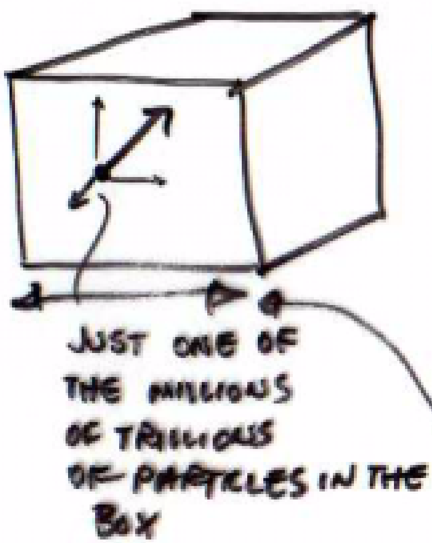
IF YOU KNOW ANY 3 OF P, V, N, T - YOU CAN FIND OUT THE 4TH ONE.

☐ So, why does this law work?

$PV = k N T$ — kind of like Kepler's laws
- striking regularity
- must be explained
- Newton's laws (developed for planets!) can do it!

MODEL: A gas is composed of point-like particles, each particle can only collide with its neighbors ~~to~~ via energy-conserving collisions – no sticking

– That's it. What does Newton say?

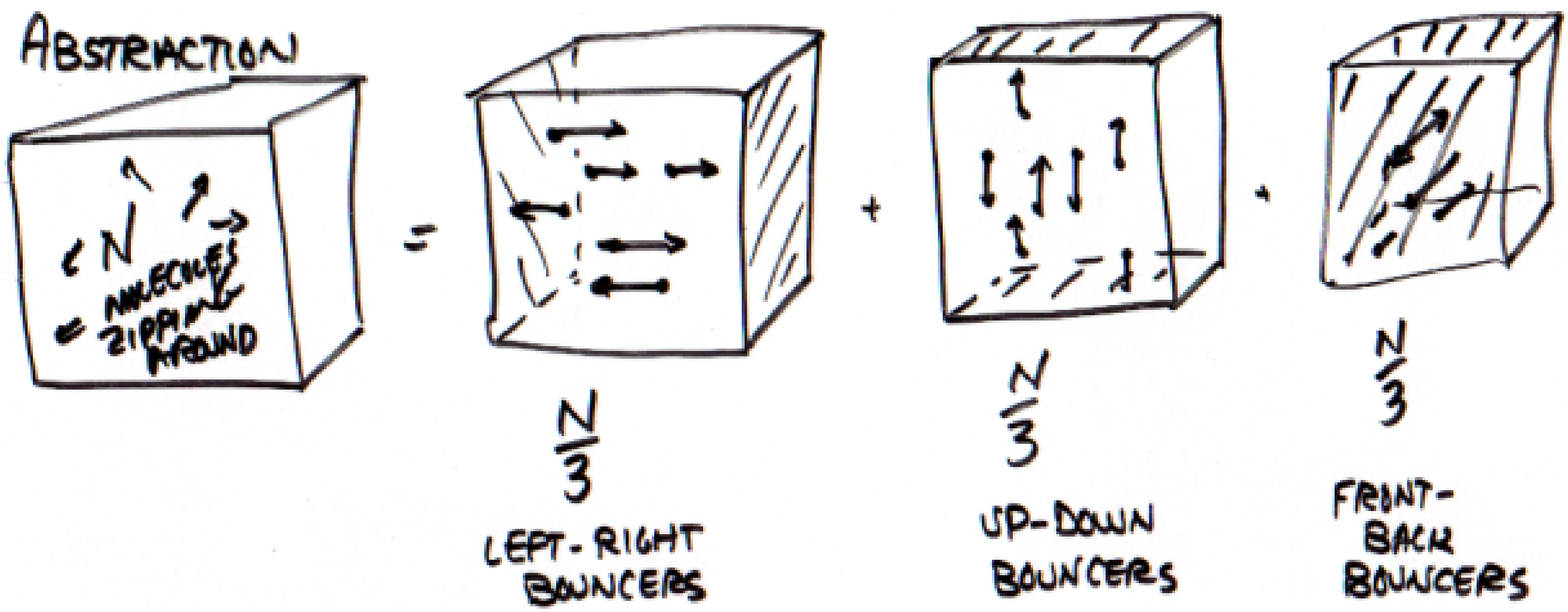


Moves in 3 dimensions.

If motion is trully random, part. could equally well be travelling in <u>ANY</u> direction.

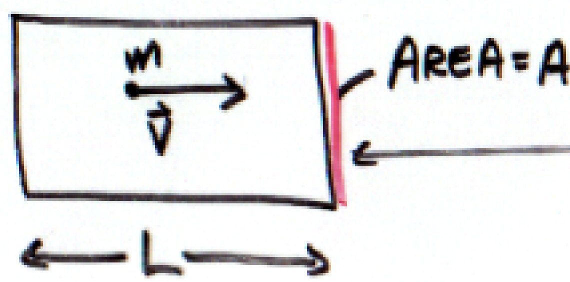
Lets say it is going exactly back + forth.

☐ ABSTRACTION



IF MOLECULAR MOTION IS TRULLY RANDOM, THE EFFECT IS THE SAME. CONSIDER JUST THE LEFT-RIGHT BOUNCERS. EACH ONE CAN OPERATE ALL BY ITSELF.

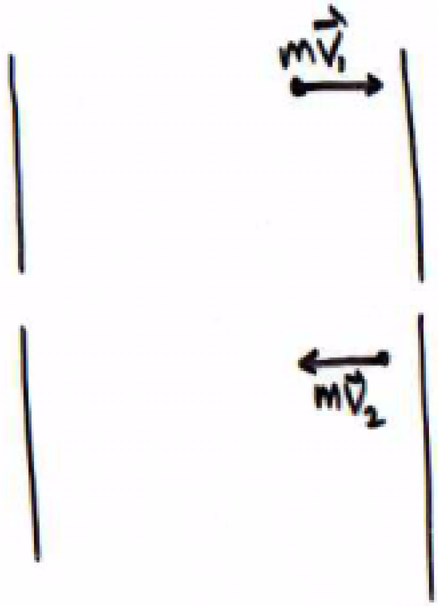
DEFINITE:



WHAT IS THE EFFECT ON THIS WALL, FROM THE COLLISION OF JUST ONE PARTICLE?

BASIC MECHANISM IS JUST REFLECTION!

☐ ONE PARTICAL:



t_1 MOMENTUM $= m\vec{V}_1 = mV$ IN $+X$ DIRECTION

t_2 MOMENTUM $= m\vec{V}_2 = -mV$ IN X DIRECTION.

CHANGE IN MOMENTUM: $\Delta P = (m\vec{V}_2 - m\vec{V}_1) = -2mV$

$\therefore$ (NEWTON'S 2ND LAW) A FORCE ACTED ON THE PARTICLE.

$$\Delta t\, F_{\text{ON MOLECULE}} = -2mV.$$

NEWTON'S 3RD LAW; THE FORCE ON THE WALL IS EQUAL + OPPOSITE $\Delta t\, F_{\text{WALL}} = -\Delta t\, F_{\text{MOLECULE}} = 2mV$ IN $+X$ DIRECTION

Δt IS THE PROBLEM!

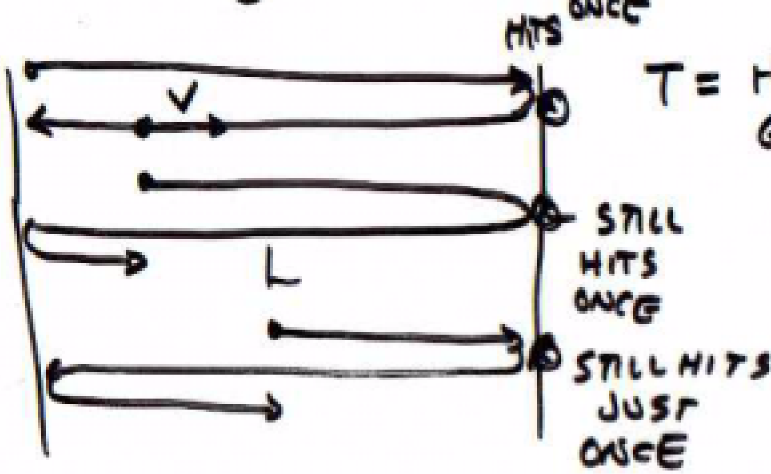
☐ Δt IS HOW MUCH TIME FOR THE COLLISION TO HAPPEN!

HOW COULD WE EVER KNOW THIS?

ONE WAY AROUND IT:

FOR EACH COLLISION, WALL HAS TO SUPPLY $-2mv$ IN MOMENTUM.

HOW MUCH TIME DOES IT TAKE FOR EACH + EVERY OF THE $\frac{N}{3}$ MOLECULES TO HIT THE WALL EXACTLY ONCE?



T = HOW MUCH TIME TO GO FULL LENGTH OF BOX TWICE?

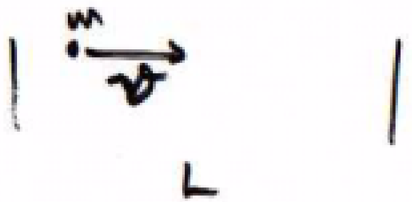
$$T = \frac{2L}{V}$$

$\therefore$ TOTAL FORCE ON WALL OF BOX $= \left(\frac{\text{MOMENTUM}}{\text{PARTICLE}}\right) \frac{1}{T} \cdot$ # OF PARTICLES

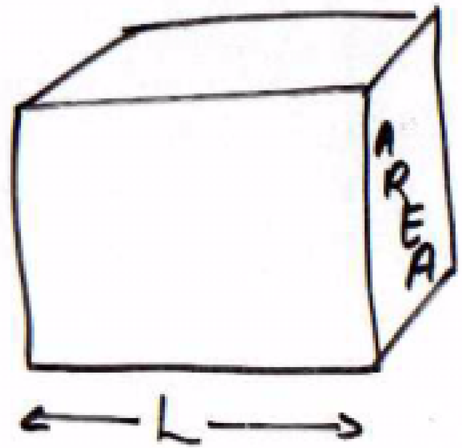
☐ SAY IT ANOTHER WAY:

$$\underbrace{T}_{\text{IN THIS TIME EACH HITS \underline{ONCE}}} \cdot F_{WALL}^{TOTAL} = \frac{\text{MOMENTUM}}{\text{PARTICLE}} \cdot \#\text{ OF PARTICLES}$$

GIVING THIS MUCH MOMENTUM



$$T = \frac{2L}{v} \; ; \; \frac{\text{MOM}}{\text{PART}} = 2mv$$



VOLUME = L · AREA

SO $\left(\frac{2L}{v}\right) \cdot F_{WALL} = 2mv \cdot \frac{N}{3}$

VELOCITY NOT VOLUME!

OR; $F_{WALL} = \frac{N}{3} \frac{mv^2}{L}$

$$\boxed{\frac{F_{WALL}}{AREA} = \frac{N}{3} \frac{mv^2}{L \cdot AREA}}$$

VERY CLOSE TO WHAT WE'RE AFTER

☐ SO FROM NEWTON (+ RANDOM VELOCITY OF THE GAS PARTICLES)

$$\frac{F_{WALL}}{\text{AREA WALL}} = \frac{N}{3} \cdot \frac{mv^2}{L \cdot \text{AREA}} = \frac{2N}{3} \cdot \left(\tfrac{1}{2} mv^2\right) \cdot \frac{1}{\text{VOLUME}}$$

$\text{AREA WALL} \equiv \text{PRESSURE}$

$$\text{PRESSURE} = \frac{2}{3} \cdot \frac{(\text{NUMBER OF PART.}) \times (\text{KINETIC ENERGY OF EACH PART.})}{\text{VOLUME}}$$

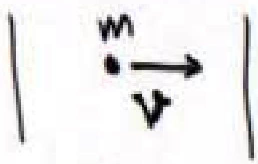
OR $(\text{PRESSURE}) \times (\text{VOLUME}) = \frac{2}{3}\,(\text{NUMBER}) \times (\text{ENERGY OF EACH PART})$

$$P \quad V \quad = k \quad N \quad T$$

TEMPERATURE MAKES KINETIC ENERGY GET BIG.
A GAS HAS A BUNCH OF K.E. STORED.

☐ KINETIC THEORY:

$$PV = kNT$$ - IDEAL GAS LAW



$$PV = \frac{2}{3} N \left(\frac{1}{2} m v^2\right)$$ KINETIC THEORY

PARTICLES BOUNCING BACK + FORTH,
KINETIC ENERGY
$= \frac{1}{2} m v^2$

BLUE BITS: $kT = \frac{2}{3} \cdot \left(\frac{1}{2} m v^2\right)$

THE TEMPERATURE TELLS US THE KINETIC ENERGY OF EACH GAS PARTICLE.

PREDICTION: IF YOU RAISE THE TEMPERATURE BY 1°K,
THE ENERGY IN THE GAS INCREASES:

ENERGY IN GAS = (NUMBER OF PARTICLES) × (ENERGY/PART.)

$= (N) \cdot \left(\frac{1}{2} m v^2\right)$

- NO MATTER THE GAS.

☐ EXAMPLE — EXAM STYLE.

~~AVOGADRO~~
AVOGADRO'S LAW: IF $P = 1$ ATMOSPHERE (≈ 30 IN MERCURY ETC. ETC.)

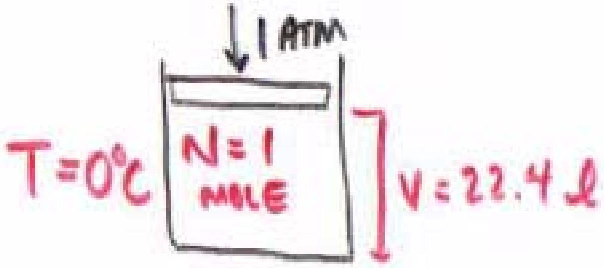
AND $T = 0°C$ (FREEZING POINT OF WATER)
$= 272°K$ ←

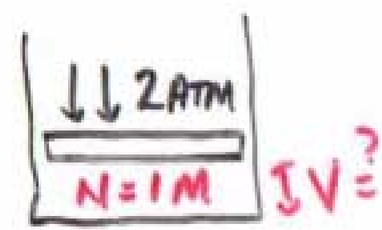
AND $V = 22.4$ LITERS (LITER = $1000\ cm^3$)

THEN $N = 1$ MOLE ; 1 MOLE = 6.02×10^{23} — JUST A NUMBER!

QUESTION: YOU HAVE 22.4 LITERS OF AIR, HELD AT 0°C UNDER 1 ATM OF PRESSURE.

HOLDING THE TEMP. FIXED, SUPPOSE YOU INCREASE PRESSURE TO 2 ATM. HOW MUCH VOL. DOES THE GAS TAKE UP NOW?





T=0°C
=272°K

$PV = K \cdot N \cdot T$

$V = \frac{K \cdot N \cdot T}{P}$

$V = \boxed{\frac{(K \cdot N \cdot T)}{1\ ATM}} \cdot \frac{1\ ATM}{P}$

22.4 ″ *

$V = 22.4\ell \cdot \frac{1\ ATM}{2\ ATM} = \frac{22.4\ell}{2} = 11.2\ell$

VOL GETS CUT BY 1/2.