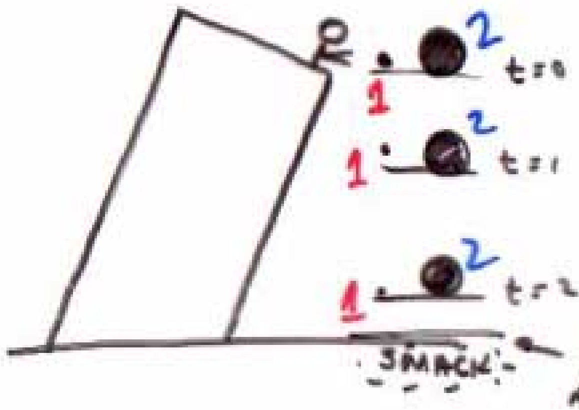
☐ FALLING OBJECTS: WHAT DO THE LAWS TELL US?

OBSERVED FACT: ALL OBJECTS, REGARDLESS OF SIZE, SHAPE, ETC. **ACCELERATE** TOWARDS THE GROUND, WITH AN ACCELERATION = $9.8 \frac{\text{METER}}{\text{SEC}^2} = g$ ← DEFINING A SYMBOL.

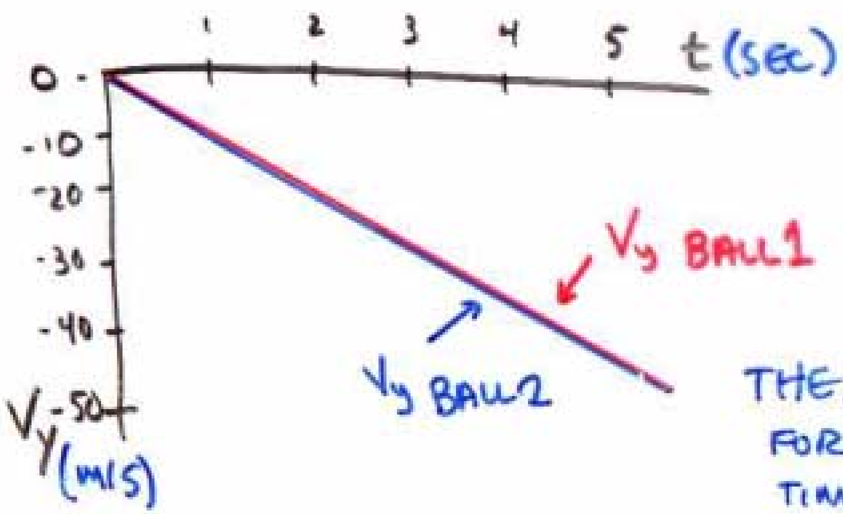
→ WE'LL USE $g = 10 \frac{\text{METER}}{\text{SEC}^2}$ WHEN WE NEED A NUMBER.

"PISA" EXPERIMENT



SAME VELOCITY VECTOR FOR BIG + SMALL BALL.

HIT AT SAME TIME.



THE SAME FOR ALL TIME, t

$$V_y = -10 \frac{m}{s^2} \cdot t$$
$$= -g \cdot t$$

V_y CHANGES ALL THE TIME.

☐ MINI EXPT. #4 (5 POINTS ON EXAM #2)

THE PISA EXPERIMENT. DO OBJECTS OF DIFFERENT MASS FALL AT THE SAME RATE?

GET 2 OBJECTS OF DIFFERENT MASS ... 2 BALLS, PEN, RULER WHATEVER.

PICK SOME CONVENIENT HEIGHT (1 YARD, 2 FEET, 6 FEET)

+ DROP OBJECTS: ~~Rec~~ $\boxed{\text{RECORD WHICH ONE HITS FIRST}}$ - DO THIS 3 TIMES

THEN PICK ANOTHER HEIGHT ... + DROP THEM AGAIN, 3 TIMES

HEIGHT	TRIAL 1	TRIAL 2	TRIAL 3
1 FT	BIG	BIG	SMALL
3 FT	SMALL	SMALL	SMALL
6 FT	BIG	BIG	BIG

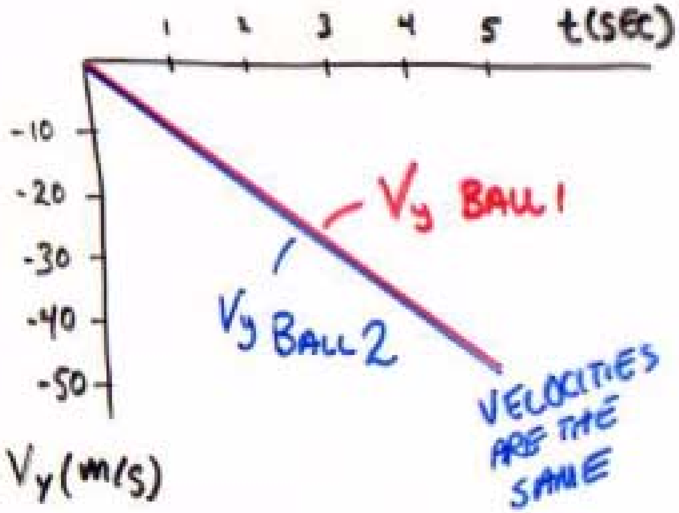
DATA TABLE.

~~KG RATHER~~ ~~Does~~ WHICH OBJECT FALLS FASTEST BASED ON YOUR DATA?

☐ FALLING OBJECTS, CONTINUED

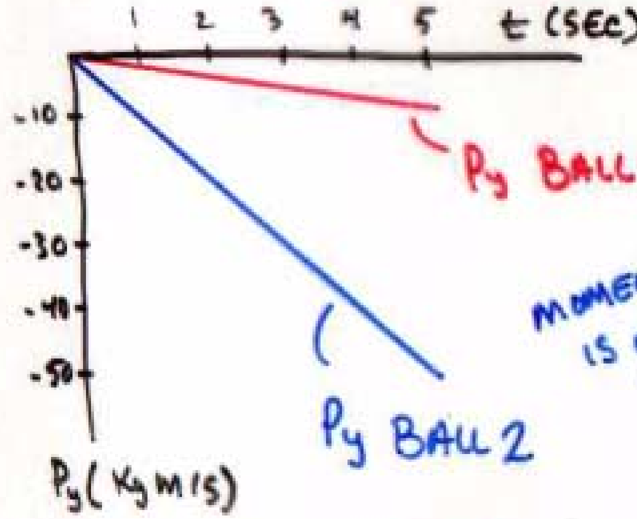
WHAT DOES THIS HAVE TO DO WITH THE VECTOR MOMENTUM?

VELOCITIES



MOMENTUM: $\vec{P} = m \cdot \vec{V}$

LET $m_1 = \frac{1}{5}$ Kg $m_2 = 1$ Kg



MOMENTUM OF 2 IS MUCH BIGGER THAN THAT OF 1

$m_1 < m_2$.

$$\boxed{\begin{aligned} V_y &= -g\,t \\ &= -\frac{10\ \text{METER}}{\text{SEC}^2} \cdot t \end{aligned}}$$

SAME FOR 1 AND 2

- YOU GET HURT BY MOMENTUM, NOT VELOCITY.

$$P_y = -m_1 g t = -\frac{1}{5}\text{Kg} \cdot 10\,\frac{\text{METER}}{\text{S}^2} \cdot t$$

$$= -\left(2\,\frac{\text{Kg}\cdot\text{m}}{\text{SEC}}\right) \cdot \left(\frac{t}{\text{SEC}}\right) \text{ - BALL 1}$$

$$P_y = -m_2 g t = -\frac{10\,\text{Kg}\,\text{m}}{\text{SEC}} \cdot \left(\frac{t}{\text{SEC}}\right) \text{ BALL 2}$$

BOTH MOMENTA CHANGE WITH TIME.

☐ FALLING OBJECTS CONTINUALLY CHANGE THEIR MOMENTUM.

LAW 1: AN OBJECT "FREE OF FORCES" KEEPS ITS MOMENTUM UNCHANGED.

Q: ARE FALLING OBJECTS "FREE OF FORCES"?

LAW 2: IF A FORCE $\vec{F}$ ACTS ON A BODY OF MASS m DURING A TIME Δt, THEN THE OBJECT'S MOMENTUM CHANGES:

CHANGE IN MOMENTUM = FORCE × Δt

$\xrightarrow{\text{DEFINITIONS}}$

$$\Delta \vec{P} = \vec{F} \Delta t$$

$$(m\vec{V}_f - m\vec{V}_i) = \vec{F} \cdot (t_f - t_i)$$ – JUST DEFINITIONS.

$$\boxed{m\left(V_{y_{FINAL}} - V_{y_{INITIAL}}\right) = F_y\left(t_{FINAL} - t_{INITIAL}\right)}$$

NOT THERE YET, BUT CLOSE –

DROP AN OBJECT AT TIME t ... VELOCITY INCREASES $V_y = -g \cdot t$

☐ THE GRAVITATIONAL FORCE:

RECAP: P_y CHANGES CONTINUALLY: $\boxed{P_y = -mgt}$

SO A FORCE MUST BE CHANGING P_y:

2ND LAW — $\boxed{\Delta \vec{P} = \vec{F} \Delta t}$

$$P_y(t_{FINAL}) - P_y(t_{INITIAL}) = F_y \cdot (t_{FINAL} - t_{INITIAL})$$

$$-mg \cdot (t_{FINAL}) - (-mg \cdot t_{INITIAL}) = F_y \cdot (t_{FINAL} - t_{INITIAL})$$

$$-mg\, t_{FINAL} + mg\, t_{INITIAL} = F_y\,(t_{FINAL} - t_{INITIAL})$$ — SOLVE THIS FOR F_y THE UNKNOWN FORCE

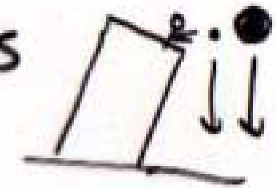
$$\rightarrow F_y = \frac{-mg\, t_{FINAL} + mg\, t_{INITIAL}}{t_{FINAL} - t_{INITIAL}}$$

$$= mg \cdot \frac{(t_{INITIAL} - t_{FINAL})}{(t_{FINAL} - t_{INITIAL})}$$

$$= mg \cdot \frac{\cancel{(t_{INITIAL} - t_{FINAL})}}{-1 \cdot \cancel{(t_{INITIAL} - t_{FINAL})}} = \boxed{-mg = F_y = \text{FORCE OF GRAVITY}}$$

☐ LINE OF REASONING:

1. GALILEO AT PISA SHOWED THAT ALL FALLING OBJECTS ACCELERATE AT THE SAME RATE.



2. ACCELERATION MEANS A CHANGE IN VELOCITY.

3. A CHANGE IN VELOCITY MEANS A CHANGE IN MOMENTUM.

4. CHANGING MOMENTUM MEANS THERE MUST BE A FORCE.
   - 1ST LAW OF NEWTON.

5. THE FORCE IS PROPORTIONAL TO THE MASS (GRAVITY ONLY!)
   - 2ND LAW OF NEWTON.

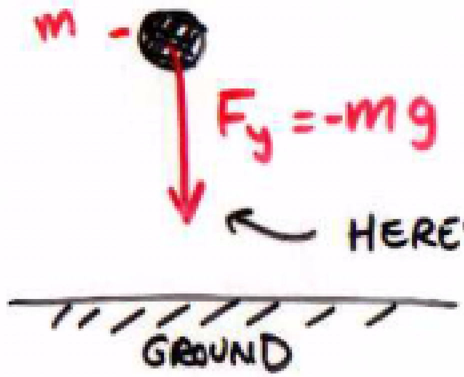
$$\vec{F}_{GRAV} = -mg$$ - DOWNWARD.

YOU SOMETIMES CALL THIS THE "WEIGHT" OF AN OBJECT.
THE MORE THE MASS THE MORE THE WEIGHT,
EVEN THOUGH EVERYTHING FALLS AT THE SAME RATE

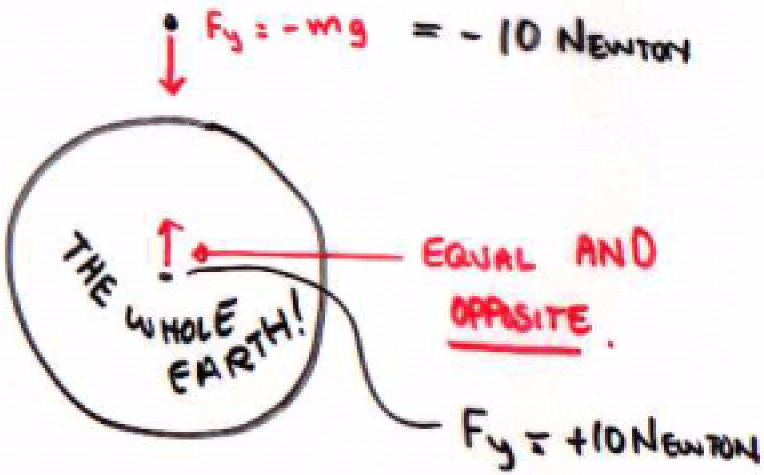
11-10

11-12

☐ WHAT ABOUT THE 3RD LAW?



← HERE'S A FORCE... WHERE'S ITS PAIR?



EXAMPLE: $m = 1\,kg$

SUPPOSE FALLS FOR 1 SEC.

FORCE ON EARTH = 10 NEWTON

DURATION = $\Delta t = 1$ SEC.

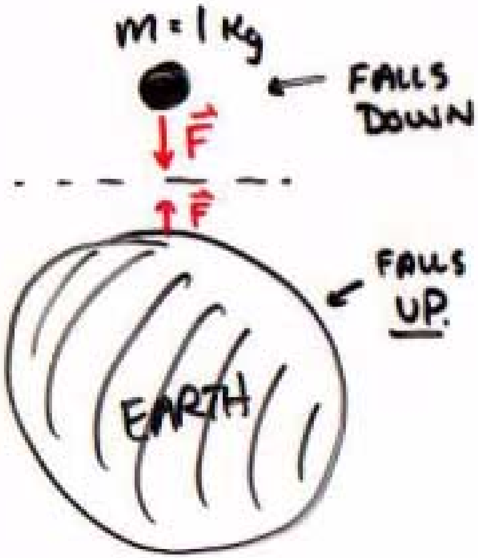
$\Delta P_{EARTH} = 10\,\text{NEWTON} \cdot 1\,\text{SEC}$

$= 10\,\dfrac{\text{kg meter}}{\text{SEC.}}$

$$\boxed{\Delta P_{EARTH} = M_{EARTH} \cdot \Delta V_{EARTH} = 10\,\frac{kg\,m}{sec}}$$

HOW FAST DOES EARTH MOVE? SOLVE FOR ΔV_{EARTH}

☐ Again, how fast does the Earth move?



$$\Delta P_{EARTH} = \boxed{m_{EARTH} \cdot \Delta V_{EARTH} = 10 \frac{\text{Kg meter}}{\text{SEC}}}$$

SOLVE FOR ΔV_{EARTH}:

$$\Delta V_{EARTH} = \frac{10 \text{ Kg meter}}{\text{SEC}} \cdot \frac{1}{m_{EARTH}}$$

$$m_{EARTH} = 24 \times 10^{22} \text{ Kg} \text{ ! BIG!}$$

$$\Delta V_{EARTH} = \frac{10 \text{ meter}}{\text{SEC}} \cdot \frac{1 \text{ Kg}}{24 \times 10^{+22} \text{ Kg}}$$

$$= 40 \; 10^{-24} \frac{\text{meter}}{\text{SEC}}.$$

AT THAT RATE IT WOULD TAKE THE EARTH 800,000 YEARS TO MOVE "UP" A DISTANCE THE SIZE OF A HYDROGEN ATOM – NO WONDER THE EARTH "FEELS" SOLID.

☐ QUESTION… BIG QUESTION.

WHAT IS THE DIFFERENCE BETWEEN THE 1kg SLUG — AND THE EARTH?

DOES "GRAVITY" ONLY "COME" FROM "BIG" OBJECTS?

THE SLUG + THE EARTH ARE EXACTLY THE SAME.

THE EARTH ATTRACTS THE SLUG,

THE SLUG ATTRACTS THE EARTH —

$\vec{F}_1 = -\vec{F}_2$ … EQUAL AND OPPOSITE.

IF m_2 IS THE EARTH…

DOUBLING m_1 DOUBLES THE FORCE.

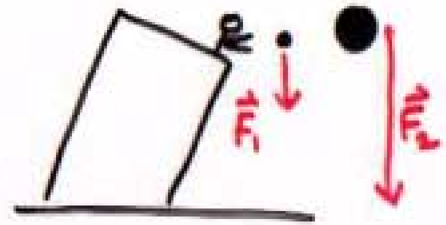
$$F = c \cdot m_1$$ — BUT ALSO IF YOU DOUBLE m_2

$$= c \cdot m_2$$

$$\boxed{F_{GRAV} = \cancel{\text{const}}\ \# \cdot m_1 \cdot m_2}$$ — HAS TO BE IF SLUG + EARTH ARE THE SAME

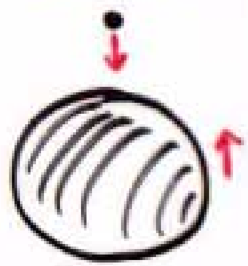
☐ REASONING AGAIN...

1. GALILEO AND THE PISA EXPERIMENT
   AND NEWTON'S LAWS 1 AND 2
   SAY THAT AN OBJECT NEAR THE SURFACE
   OF THE EARTH EXPERIENCES A FORCE:



$F_2 > F_1$, BUT ACCELERATION IS THE SAME.

2. THE 3RD LAW SAYS THERE IS AN UPWARD
   FORCE ON THE EARTH DUE TO THE BALL!



$|\text{FORCE}| = (m_{\text{BALL}}) \cdot g$

3. THERE IS NO DIFFERENCE BETWEEN EARTH + THE SLUG!

∴ F_{GRAV} BETWEEN 2 BODIES IS PROPORTIONAL TO

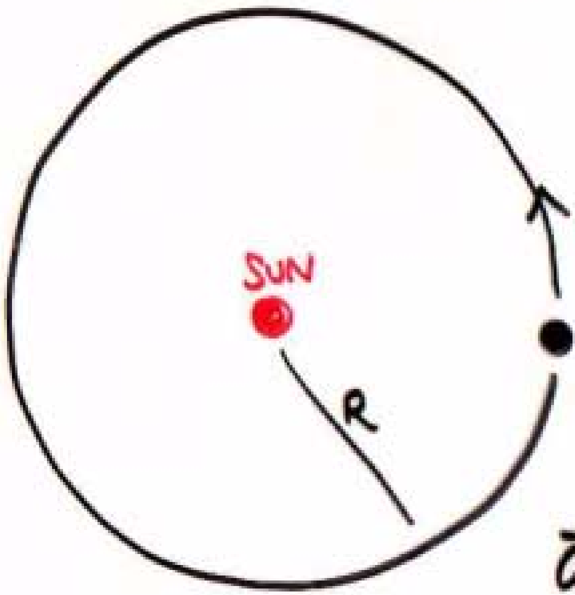
$m_1 \cdot m_2$ – PRODUCT OF THE MASSES.

$$\boxed{F_{\text{GRAV}} = (\text{SOMETHING}) \cdot m_1 \cdot m_2}$$ – GRAVITY IS UNIVERSAL.

☐ KEPLER'S LAWS FINISH THE STORY

KEPLER'S 3RD LAW: $\underbrace{T}_{\text{PERIOD OF ORBIT}}{}^{2} \doteq \underbrace{R}_{\text{RADIUS OF ORBIT}}{}^{3}$

WORKS FOR
- MERCURY
- VENUS
- EARTH
- MARS
- JUPITER
- SATURN



JUPITER

TAKES T YEARS TO GO ALL THE WAY AROUND.

$\vec{V}_{JUPITER}$ CONTINUALLY SPINS AROUND THE CIRCLE..

$$\vec{a} = (\text{INWARD})\ \frac{V^2}{R} \qquad V = \frac{2\pi R}{T}$$

$$= (\text{INWARD}) \cdot \frac{4\pi^2 R^2}{T^2 R} = (\text{INWARD})\ 4\pi^2 \frac{R}{T^2}$$

2ND LAW ... THERE IS A FORCE CAUSING THIS ACCELERATION: F_{GRAV}

$$F_{GRAV} = m_{SUN} \cdot m_{JUPITER} \cdot \underbrace{f(R)}_{\text{UNKNOWN}} = m_{JUPITER} \times \left(4\pi^2 \frac{R}{T^2}\right)$$

BUT WHEN WE FIGURE IT OUT, WE'LL KNOW **ALL THERE IS TO KNOW ABOUT GRAVITY**

☐ CONTINUE ... $F_{GRAV} = m_{JUPITER} \cdot m_{SUN} \cdot f(R)$

CIRCULAR ACCELERATION ⟶

$$m_{SUN} \cdot \cancel{m_{JUPITER}} \cdot f(R) = \cancel{m_{JUPITER}} \cdot \left(4\pi^2 \frac{R}{T^2}\right)$$

So $\boxed{f(R) = \frac{4\pi^2 R}{m_{SUN} T^2}}$

KEPLER SAYS THAT $\boxed{T^2 = C \cdot R^3}$

EXPERIMENTAL FACT.

$$f(R) = \frac{4\pi^2 R}{m_{SUN} \cdot (C \cdot R^3)} = \frac{4\pi^2 R}{(\text{CONSTANT}) \cdot R^3}$$

$$\boxed{f(R) = \frac{4\pi^2}{(\text{CONSTANT})} \cdot \frac{1}{R^2}}$$

ALL TOGETHER:

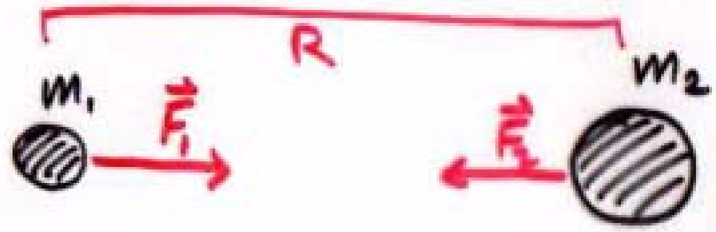
$$\boxed{F_{GRAV} = \frac{m_{JUPITER} \cdot m_{SUN}}{R^2} \cdot G}$$

UNIVERSAL CONSTANT

FOR WHOLE ENCHILADA

☐ NEWTON'S UNIVERSAL LAW OF GRAVITATION

EVERY 2 OBJECTS IN UNIVERSE ATTRACTS :



MAGNITUDE $|F_1| = |F_2| = G \dfrac{m_1 m_2}{R^2}$

"LENGTH OF VECTOR ARROWS"

— IF YOU LIVED IN 1780, YOU WOULD HAVE GRASPED EXTREME CUTTING EDGE OF SCIENTIFIC KNOWLEDGE.

☐ STRENGTH OF THE GRAVITY FORCE:

CAVENDISH – ENGLISH PHYSICIST ≈ 1781

MEASURE THE FORCE

EXPERIMENT IS ONLY WAY TO FIND OUT.

$$F = \frac{m_1 m_2}{R^2} \cdot G \, ,$$

WHERE G IS AN UNKNOWN "CONSTANT OF THE UNIVERSE"

SAME NUMBER HERE, JUPITER, 1000 YEARS AGO

EVERYWHERE IN THE UNIVERSE.

<u>HAS</u> TO BE MEASURED (… FOR NOW …)

$$F = \frac{(1\,Kg)(1\,Kg)}{(1\,meter)^2} \cdot G = \underline{6.6 \times 10^{-11} \text{ Newton}}$$

MEASURED BY CAVENDISH

(= 100 BILLION TIMES SMALLER THAN WEIGHT OF EACH BALL !)

EXTREMELY SMALL FORCES –

<u>VERY</u> HARD TO MEASURE.

☐ How small is the gravity force?

Gravity force between you (at **birth**!) and Jupiter!

$m_{BABY} = 5\,kg$

$m_{JUP} = 2\times10^{27}\,kg$

Big!

$$F_{GRAV} = \frac{m_{BABY}\cdot m_{JUP}}{R^2}\cdot G = G\frac{(5)(2\times10^{27})kg^2}{(8\cdot10^{11})^2 m^2} \quad 6\times10^{-11}\,N\frac{m^2}{kg^2}$$

$$= \boxed{10^{-6} N}$$

Tiny force

One millionth of a Newton

Suppose you were delivered by a fat doctor:

$m_{DOCTOR} = 100\,kg \quad m_{BABY} = 5\,kg$

And the doctor holds you $\frac{1}{10}$ m from his chest:

$$F_{GRAV} = \frac{(5\,kg)(100\,kg)}{(\frac{1}{10}\text{ meter})^2}\cdot 6\times10^{-11}\,N\frac{m^2}{kg^2} = 5\cdot100\cdot100\cdot6\times10^{-11}\,N$$

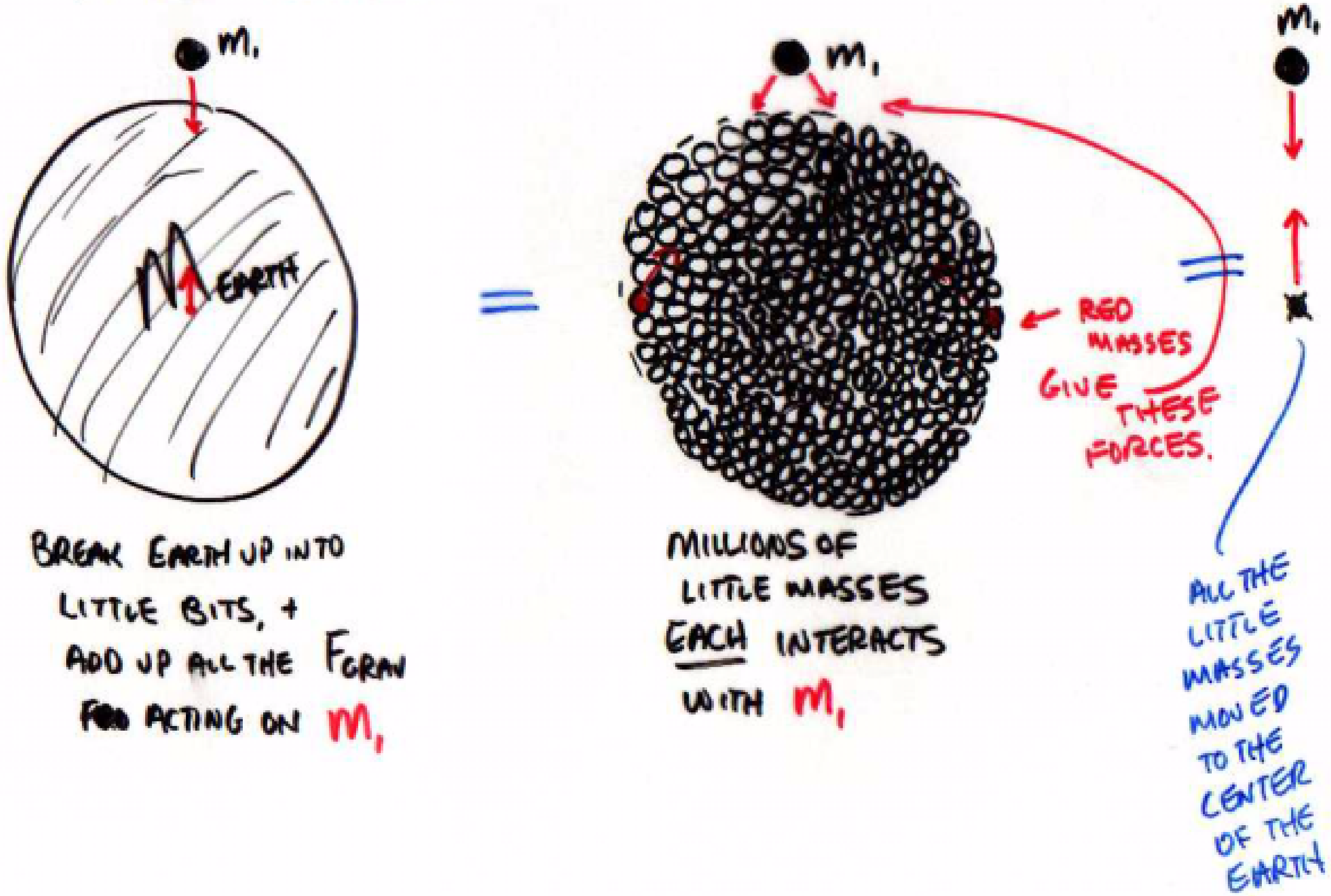
$$= \boxed{3\times10^{-6} N}$$

Small, but 3 times bigger than the gravity due to Jupiter!

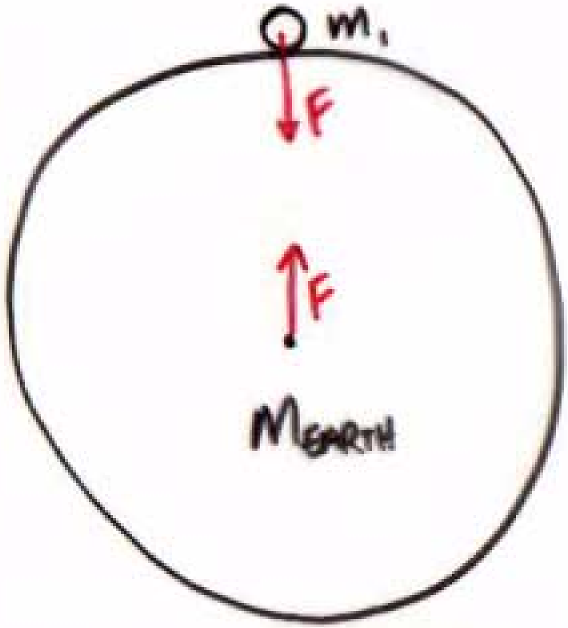
☐ How does g arise from Newton's Laws + Universal Gravitation?

11-F

The Earth "acts" as if all of its mass is concentrated at the center...



☐ SO… WHAT ABOUT g? LET AN OBJECT m_1 BE ON THE SURFACE OF THE EARTH.



NEWTON'S GRAVITY LAW:

$$F_1 = \frac{m_1 \cdot M_{EARTH}}{(R_{EARTH})^2} \cdot G$$

… NUMBERS

$$= \frac{m_1 \cdot (6\times10^{24}\,\mathrm{Kg})}{(6\times10^{6}\,\mathrm{m})^2} \cdot 6\times10^{-11} \frac{\mathrm{N\,m^2}}{\mathrm{Kg^2}}$$

$$= m_1 \cdot \frac{6\times10^{24} \cdot 6\cdot10^{-11}}{6\cdot10^{+6}\cdot 6\times10^{+6}} \frac{\mathrm{N}}{\mathrm{Kg}}$$

$$= \frac{m_1}{\mathrm{Kg}} \cdot \frac{36\times10^{+13}}{36\times10^{12}} \mathrm{N} = \left(\frac{m_1}{\mathrm{Kg}}\right) \cdot 10\,\mathrm{N}$$

$$= m_1 \cdot 10 \frac{\mathrm{N}}{\mathrm{Kg}} = m_1 \cdot 10 \left(\frac{\mathrm{Kg\,m}}{\mathrm{s^2}} \frac{1}{\mathrm{Kg}}\right)$$

$$= m_1 \cdot 10 \frac{\mathrm{m}}{\mathrm{s^2}} = \boxed{m_1\, g}$$

g COMES ABOUT DIRECTLY FROM

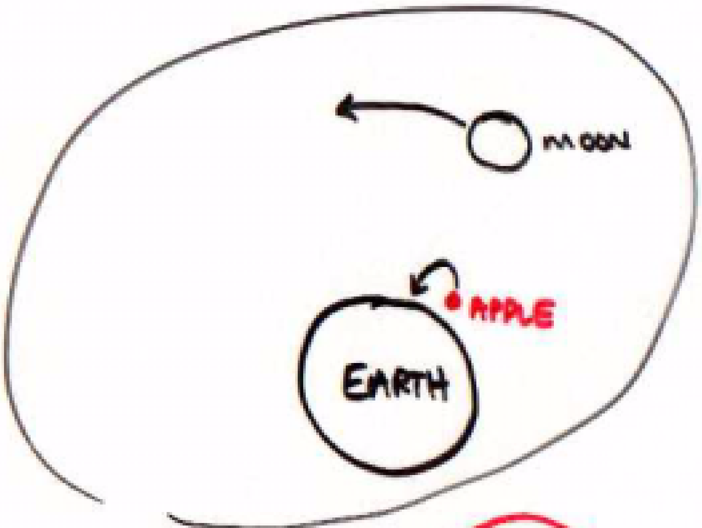
$$F = \frac{m_1 m_2}{R^2} G$$

THE EARTH IS MASSIVE BUT IT IS ALSO

FAR AWAY

11-G

☐ Falling Apples, Moons, + Kepler's Laws

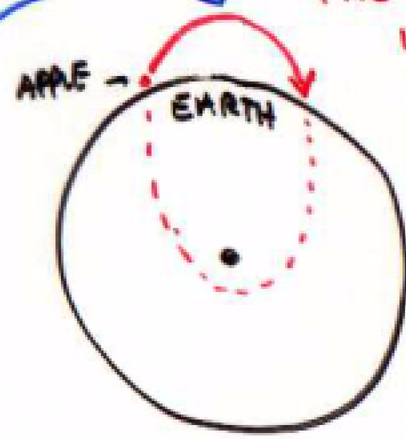


Are the motions of the Moon and the Apple different?

Moon: Covered by Kepler's 1st law
its orbit is a circle or an ellipse (oval).

Apple: Throw an apple, + its motion is curved:

This curving motion is just like that of a planet near the farthest part of its orbit!

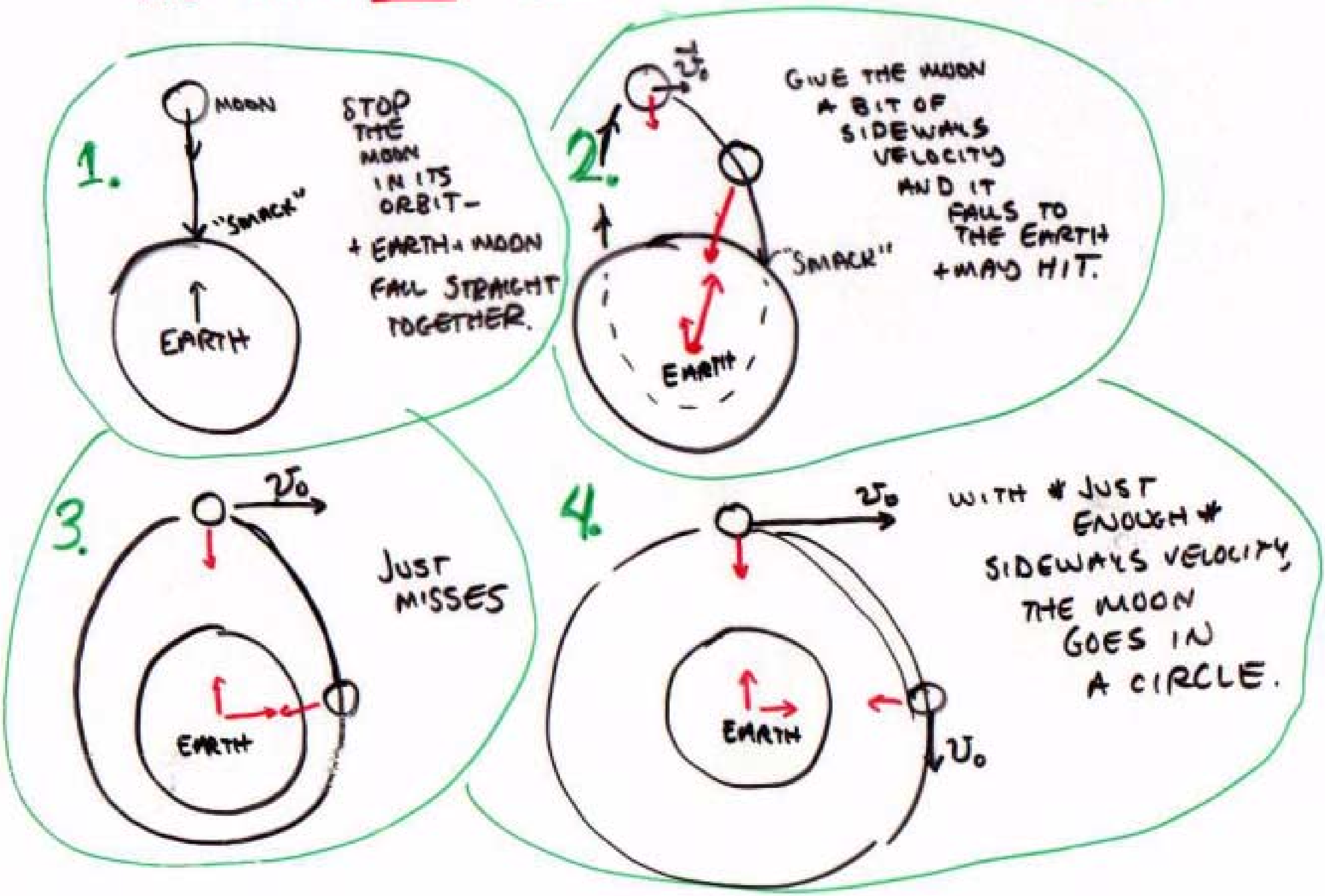


The apple "falls" in an orbit - just like a planet!

Can't see its whole motion because the Earth is in the way

☐ THE APPLE "ORBITS" JUST LIKE THE MOON, AND THEREFORE,

THE MOON **FALLS** JUST THE WAY AN APPLE DOES —

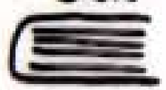


4. THE EARTH'S GRAVITATIONAL INTERACTION W/ AN OBJECT CREATES A FORCE. THE SIZE OF THE FORCE IS BIG FOR MASSIVE OBJECTS - SMALL FOR NOT SO MASSIVE OBJECTS. IF THE FORCES ARE DIFFERENT, HOW CAN THE ACCELERATION'S BE THE SAME?

PS112

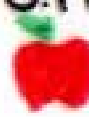
ONLY GRAVITY.

1kg BOOK



mg = 10 NEWTON

0.1 kg



APPLE

mg = 1 NEWTON

$F = m \cdot a$

$F = m \cdot a$

$F_g = \boxed{\cancel{m} \cdot g} = \cancel{m} \cdot a$

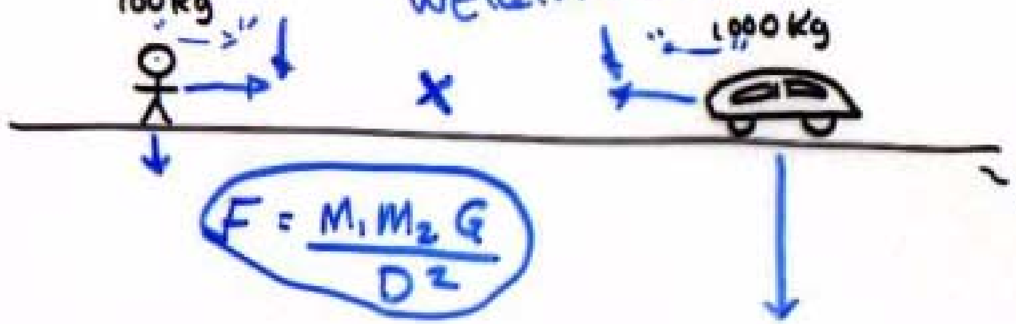
$a = g$

$\boxed{\text{FORCES}} = \boxed{M \cdot \text{ACCEL}}$

$\boxed{\cancel{m}\, g} \qquad \cancel{m}\ \text{ACCEL}$

11-9

WEIGHT = $m \cdot g$



$$F = \frac{M_1 M_2 G}{D^2}$$

"FORCE OF GRAV."

a) USE AN ARROW TO SHOW WEIGHT OF EACH OBJECT.

b) DO YOU AND THE CAR ATTRACT EACH OTHER?

DRAW THE GRAVITY FORCES ON EACH.

DOES THIS MEAN THAT EVENTUALLY YOU + THE CAR WILL SMACK TOGETHER?

3[RD] LAW: FORCE ON PERSON FROM CAR

= " " CAR FROM PERSON

SLOW.
BIG MASS

BOWLING BALL

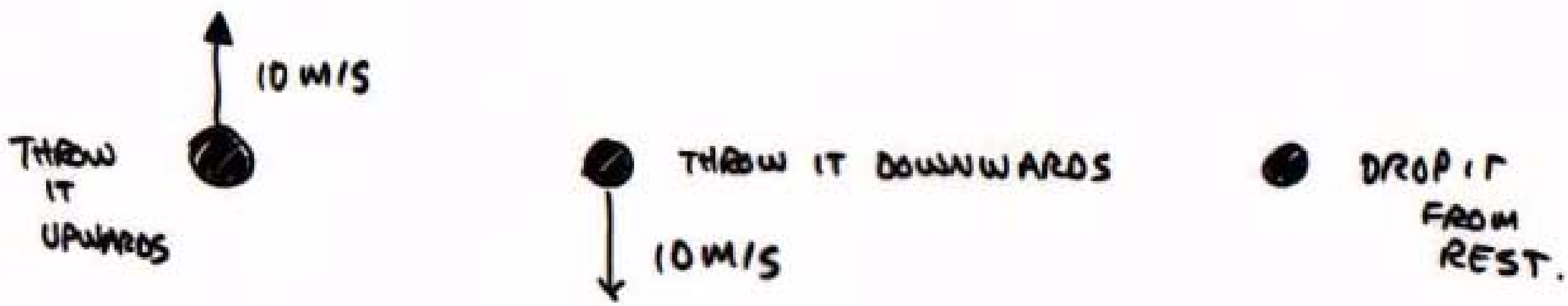
BASEBALL } FASTEST.

SMALL MASS

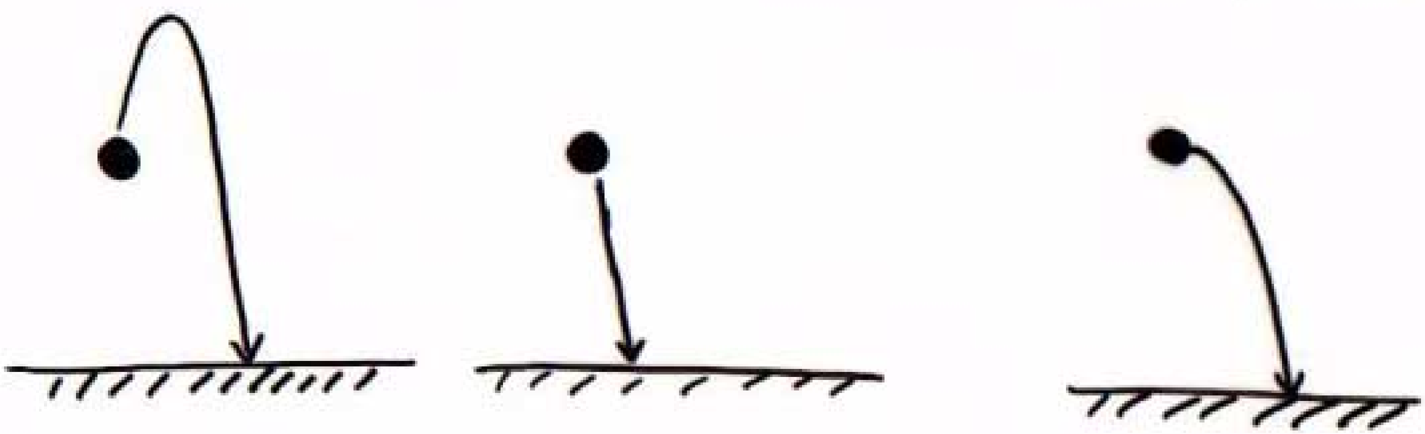
FAST -

☐ (NOT IN BOOK)

1 kg BALL

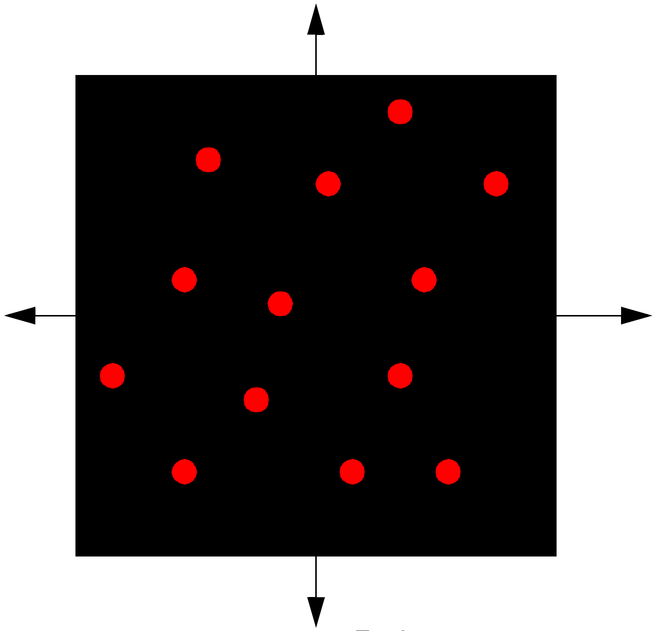


AFTER THE BALL LEAVES YOUR HAND ... WHAT FORCE IS ON IT IN EACH CASE? HOW DOES EACH ACCELERATE? WHICH HITS 1ST 2ND 3RD?

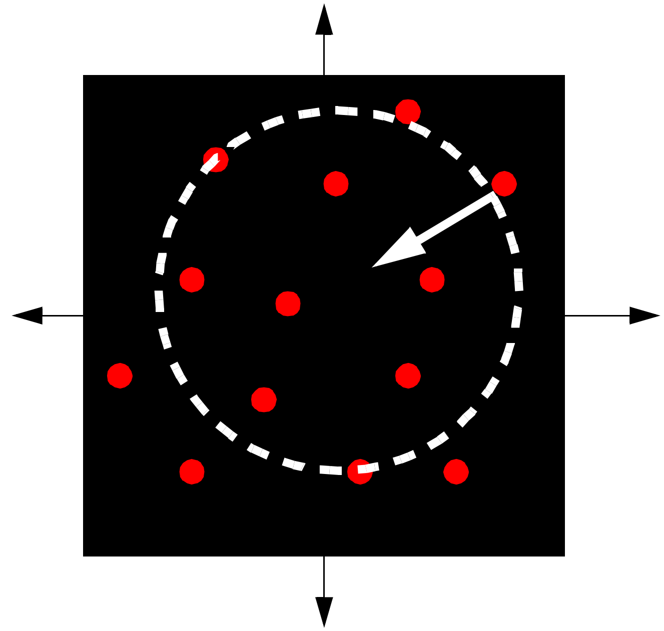


● Something Newton Missed

- What does gravity mean for the Universe?



Everything
Pulls Everthing
Balanced



Unbalanced!

- Can’t have a “Universe at Rest”
- Gravity will make the Universe COLLAPSE.
- Big Bang, Big Crunch