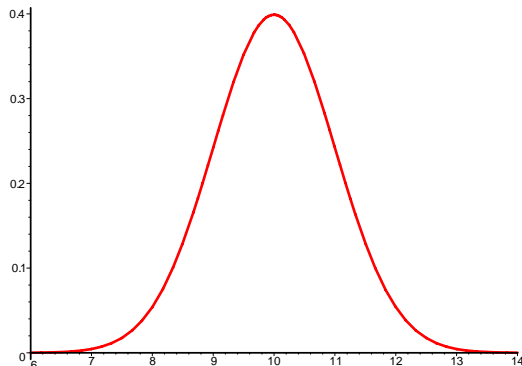


**Worksheet: Normal Random Variables**

Suppose that  $X$  is a normally distributed random variable with mean  $\mu = 10$  and standard deviation  $\sigma = 1$ .



1. Label the mean  $\mu$  on the graph. *The mean is at  $x = 10$ .*
2. What is the total area under the curve? *1*
3. Find the probability that  $X$  is less than the mean, by following the steps below.
  - a. This question is asking you to find  $P(X < 10)$ .
  - b. Find the  $z$ -score for  $x = 10$ . The formula for the  $z$ -score of  $x$  is  $z = \frac{x - \mu}{\sigma}$ .

$$z = \frac{10 - 10}{1} = 0$$

Then  $P(X < 10) = P(Z < 0)$ .

- c. Use the table to find this probability.

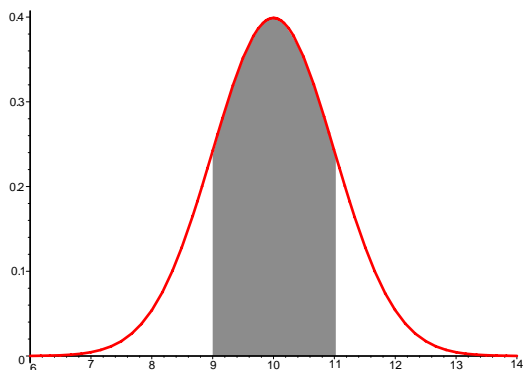
$$P(X < 10) = P(Z < 0) = 0.5$$

*Look at the picture. This answer should make perfect sense.*

4. Find the probability that  $X$  is within one standard deviation of the mean, following the steps below.

- a. This question is asking you to find  $P(9 < X < 11)$ .

Sketch the normal curve for this random variable and shade the region corresponding to this probability.



- b. Write the probability as the difference of two probabilities of the form  $P(X < c)$ .

$$P(9 < X < 11) = P(X < 11) - P(X < 9).$$

- c. Find the  $z$ -scores, and use the table to complete the problem.

*The  $z$  score for  $x = 9$  is  $(9 - 10)/1 = -1$ .*

*The  $z$ -score for  $x = 11$  is  $(11 - 10)/1 = 1$ .*

*On the table, we can find  $P(Z < -1) = 0.1587$  and  $P(Z < 1) = 0.8413$ .*

$$\begin{aligned} P(9 < X < 11) &= P(X < 11) - P(X < 9) = P(Z < 1) - P(Z < -1) \\ &= 0.8413 - 0.1587 = 0.6826. \end{aligned}$$

*The probability that  $X$  is within one standard deviation of its mean is 68.26%.*