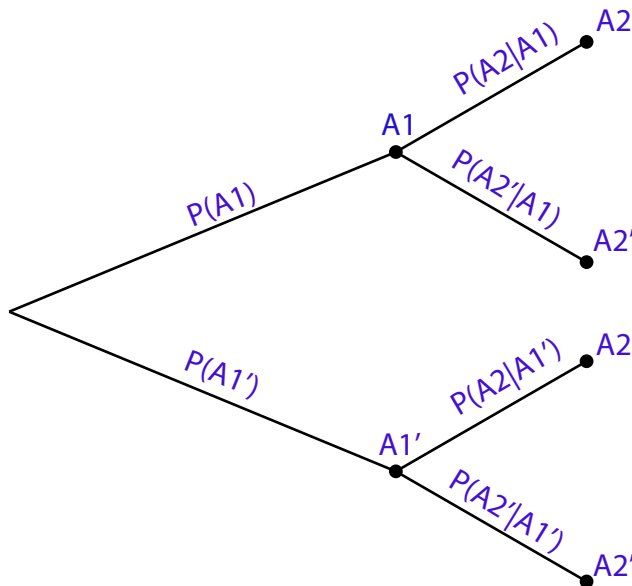


Worksheet: Tree diagrams and conditional probabilities

Consider the experiment in which two cards are drawn from a deck without replacement.
 Question: What is the probability that both cards are aces?

- A_1 = the event that the first card chosen is an ace,
- A_1' = the event that the first card chosen is not an ace,
- A_2 = the event that the second card chosen is an ace, and
- A_2' = the event that the second card chosen is not an ace.



Tree Diagram with symbols

- Label each of the nodes of the diagram with the symbol of the appropriate event. Your labels should be from this list: A_1 , A_1' , A_2 , and A_2' .
- Label each of the branches of the diagram with the appropriate probability symbols. Your labels should be from this list: $P(A_1)$, $P(A_1')$, $P(A_2|A_1)$, $P(A_2'|A_1)$, $P(A_2|A_1')$, and $P(A_2'|A_1')$.
- What is the symbol for the probability that the first card is an ace and the second card is an ace? (It will be $P(\text{something} \cap \text{something})$.)

$$P(A_1 \cap A_2)$$

- Indicate (underline or circle) the branches on the diagram above that correspond to getting an ace for both the first and second cards.
- Write down the product of the probabilities on the branches that you marked.

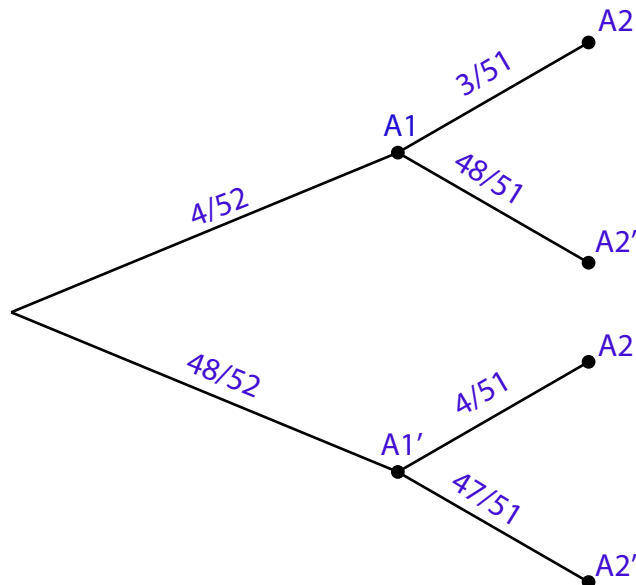
$$P(A_1)P(A_2|A_1)$$

- What does the Product Rule for Probability say this is equal to?

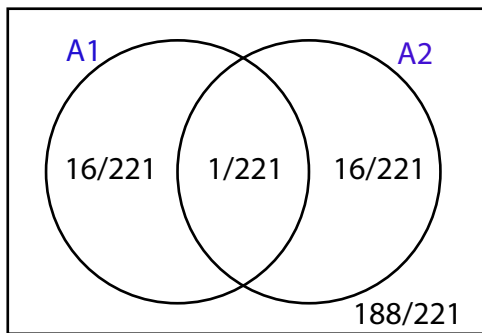
$$P(A_1)P(A_2|A_1) = P(A_1 \cap A_2)$$

Upshot: The product rule for probability says that you can multiply the numbers on the branches of the tree diagram to get the probabilities of intersections of events.

Number of cards in the deck before any cards are chosen	52
Number of aces in the deck before the first card is chosen	4
Probability that the first card is an ace	$P(A_1) = 4/52$
Number of cards that are not aces in the deck before the first card is chosen	48
Probability that the first card is not an ace	$P(A'_1) = 48/52$
Number of cards in the deck after the first card is chosen	51
Number of aces in the deck after the first card is chosen, given that the first card is an ace	3
Probability that the second card is an ace, given that the first card is an ace	$P(A_2 A_1) = 3/51$
Number of cards that are not aces in the deck after the first card is chosen, given that the first card is an ace	48
Probability that the second card is not an ace given that the first card is an ace	$P(A'_2 A_1) = 48/51$
Number of aces in the deck after the first card is chosen, given that the first card is not an ace	4
Probability that the second card is an ace, given that the first card is not an ace	$P(A_2 A'_1) = 4/51$
Number of cards that are not aces in the deck after the first card is chosen, given that the first card is not an ace	47
Probability that the second card is not an ace, given that the first card is not an ace	$P(A'_2 A'_1) = 47/51$



1. Draw a two circle Venn Diagram and label one circle A_1 and the other A_2 . Leave enough space to write the probabilities in the regions.



2. Use the information on the tree diagram to answer each of the following questions. Write both the probability symbol and the numerical answer, and then add your answers to the appropriate region in the Venn Diagram above.

- a. What is the probability that both cards are aces?

$$P(A_1 \cap A_2) = 1/221$$

- b. What is the probability that the first card is an ace and the second card is not an ace?

$$P(A_1 \cap A_2') = 16/221$$

- c. What is the probability that the first card is not an ace and the second card is an ace?

$$P(A_1' \cap A_2) = 16/221$$

- d. What is the probability that the first card is not an ace and the second card is not an ace?

$$P(A_1' \cap A_2') = 188/221$$

3. What is the probability that the second card is an ace?

$$P(A_2) = 1/221 + 16/221 = 1/13$$

4. What is the probability that the second card is not an ace? Write the symbol and numerical answer.

$$P(A_2') = 16/221 + 188/221 = 12/13$$

5. What is the probability that the first card is an ace given that the second card is an ace? Write the symbol, the formula for the conditional probability, and the numerical answer.

$$P(A_1|A_2) = \frac{P(A_1 \cap A_2)}{P(A_2)} = \frac{(1/221)}{(1/13)} = 1/17$$

6. Reality check: What is the probability that the second card is an ace given that the first card is an ace? Write the symbol and the numerical answer.

This one is on the tree diagram:

$$P(A_2|A_1) = 3/51 = 1/17$$