Homework Difference Equations

The references in this assignment refer to, "Topics from Differential and Difference Equations," by Melvin D. Lax.

I. The notation $\sum_{k=1}^{n} a_k$ stands for the sum $a_1 + a_2 + a_3 + ... + a_n$. For example, if N=3, then you could write $\sum_{k=1}^{3} a_k = a_1 + a_2 + a_3$. Table 2.1 gives formulas for certain series. In the following exercises, you will verify some of these formulas when $n = 5$, and then use the formula to find the value of the series when $n = 10$.

A. Formula: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

i. Write the series $\sum_{k=1}^{5} k$ as a sum of numbers, and then calculate the sum.

ii. Evaluate the formula $\frac{n(n+1)}{2}$ at $n = 5$. Write, "when n=5, n(n+1)/2=...." (The formula says your answers to (i) and (ii) should be the same.)

iii. Use the formula to find $\sum_{k=1}^{10} k$.

B. Formula: $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

i. Write the series $\sum_{k=1}^{5} k^2$ as a sum of numbers, and then calculate the sum.

ii. Evaluate the formula $\frac{n(n+1)(2n+1)}{6}$ at $n = 5$.

iii. Use the formula to find $\sum_{k=1}^{10} k^2$.

C. Formula: $\sum_{k=1}^{n} 1 = n$.

i. Write the series $\sum_{k=1}^{5} 1$ as a sum of numbers, and then calculate the sum.

ii. Use the formula to find $\sum_{k=1}^{10} 1$.

D. Formula: $\sum_{k=1}^{n} r^{n-k} = \frac{r^n - 1}{r - 1}$, where r is any real number except 1. (You will study this for $r = 3$.)

i. Write the series $\sum_{k=1}^{5} 3^{n-k}$ as a sum of numbers, and then calculate the sum.

ii. Evaluate the formula $\frac{r^n - 1}{r - 1}$ at $n = 5$ and $r = 3$.

iii. Use the formula to find $\sum_{k=1}^{10} 3^{n-k}$.

II. Laws of arithmetic.

A. Formula: $\sum_{k=1}^{n} ca_k = c\sum_{k=1}^{n} a_k$

i. Write the formula above when n = 5 without using summation notation (use +'s).

ii. What law of arithmetic is this?

iii. Find a formula for the series $\sum_{k=1}^{n} 3k$, using II.A. and I.A.

iv. Find a formula for the series $\sum_{k=1}^{n} 5(4^{k-1})$, using II.A. and I.C.

B. Formula: $\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$

i. Write the formula above when n = 5 without using summation notation (use +'s).

ii. What law of arithmetic is this?

iii. Find a formula for $\sum_{k=1}^{n} (k + k^2)$, using II.B., I.A, and I.B.

iv. Find a formula for $\sum_{k=1}^{n} (3 + k^3)$, using formula II.B. together with formulas from Table 2.1.

III. Theorem 2.1 says that the solution to the linear first order difference equation $y_n - ay_{n-1} = b_n$ is given by $y_n = a^n y_0 + \sum_{k=1}^{n} a^{n-k} b_k$.

A.

i. Write the first order linear equation and its solution when y_0=5, a = 1 and b_n is the series given by $b_n = 6n^3$. (Simply rewrite the above with the given a and b_n, noting that since $b_n = 6n^3$, we have $b_k = 6k^3$.

ii. Simplify the formula for y_n. Begin by using that 1^m =1.

B. Consider the linear first order difference equation given by $y_n - 9y_{n-1} = 0$, with y_0=2.

i. What are a and b_n in this equation? Find b_1, b_2 and b_3.

ii. Write the solution to this equation and simplify it.

C. Consider the linear first order difference equation given by $y_n + 9y_{n-1} = 20$, with y_0=3.

i. What are a and b_n in this equation? Find b_1, b_2 and b_3.

ii. Write the solution to this equation and simplify it.

D. Consider the linear first order difference equation given by $y_n - y_{n-1} = 5 + n^3$, with y_0=1.

i. What are a and b_n in this equation? Find b_1, b_2 and b_3.

ii. Write the solution to this equation and simplify it.