

## Homework Section 13.2 and 13.3.

- I. Do problems #22 and #28 on pages 745-746.
- II. Exponentially distributed random variables. Read problem #32 page 757. The experiment here is to choose a 55-year old African American woman at random. The random variable we are discussing is the number of years beyond 55 that she will live.
  - A. In this problem, instead of being given the parameter  $a$  in the formula for the exponentially distributed random variable, we have been given the expected value. Use the formula for the expected value  $\mu$  on page 749 to solve for the parameter  $a$ . Keep four significant figures in your value of  $a$ .
  - B. Sketch a graph of the probability distribution for the number of years beyond 55 that an African American woman will live. See Figure 10 page 749. Label the y-intercept with its value. Label the x-axis "Years beyond 55" Label the y-axis "Probability density function."
  - C. Review the blue box on the top of page 731 that tells you how to use a probability density function to calculate a probability.
    - i. What is  $P(X < 5)$ ? Write a sentence saying what  $P(X < 5)$  represents in words in addition to the numerical answer.
    - ii. What is the probability that a 55 year old African American woman will live at most 10 more years? Write the symbol for the probability in addition to the numerical answer.
    - iii. What is the probability that a 55 year old African American woman will live at most 20 more years? Write the symbol for the probability in addition to the numerical answer.
    - iv. In (ii) and (iii) you did almost the same calculations, changing only one number. Here you will replace that number with a parameter, effectively doing all such problems at once. What is the probability that a 55 year old African American woman will live at most  $t$  more years? Write the symbol for the probability as well as the formula. Your answer will be a function of  $t$ .
    - v. Write a sentence explaining how you can use the formula you found in (iv) to find the probability that a 55 year old African American woman will live at most 100 more years. (Yes, this means she would live *at most* to an age of 155. See part (vi).)
    - vi. In part (v), you should see that the exponentially distributed random variable is not a perfect model for the probabilities in this situation. Write a sentence explaining what you believe the probability that a 55-year old woman will live at most 100 more years should be. Compare this to the answer you calculated using the exponentially distributed random variable in part (v). Conclude that the model is not perfect for this situation.
- III. Normally distributed random variables.
  - A. Suppose you are given that  $Y$  is a normally distributed random variable with mean  $\mu$  and standard deviation  $\sigma$ . Let  $y$  be a given value of  $Y$ . Enumerate the steps you would take to find the probability that  $Y$  is less than  $y$ , i.e. to find  $P(Y < y)$ . (This means you should write "Step 1: ...," as if you were writing a Blue Box for the

text entitled “Finding probabilities for a normally distributed random variable.”)  
In parts (B) and (C) below you will do these steps in reverse order.

- B. Do the following for each of exercises #11-14 page 756.
- i. Sketch the graph of the standard normal distribution function, and shade in the area in question.
  - ii. Find the  $z$ -score satisfying the condition given in the exercise (i.e. do the problem).
  - iii. Denoting the standard normal random variable by  $Z$ , write the appropriate probability in symbols. For example, for #11, you should write  $P(Z < z) = 0.1$ , where  $z$  is your answer to part (ii) for #11.
- C. Read problem #30 page 758. Call the normally distributed random variable that yields the height of a pygmy  $Y$ .
- i. Sketch the graph of the probability distribution for  $Y$ , labeling the mean. Shade in the region corresponding to the middle 50% of this population. Label the lowest and highest heights of the middle 50% of the population by  $y_1$  and  $y_2$ .
  - ii. What percent of the total area is to the left of  $y_1$ ? Find the  $z$ -score  $z_1$  that corresponds to  $y_1$ . (Hint: do this like you did problems 11-14).
  - iii. What percent of the total area is to the left of  $y_2$ ? Find the  $z$ -score  $z_2$  that corresponds to  $y_2$ .
  - iv. Use the values of  $z_1$  and  $z_2$  that you found above, along with the  $\mu$  and  $\sigma$  given in the problem, to find  $y_1$ , the least height in the middle 50% of the population, and  $y_2$ , the greatest height of in the middle 50% of the population. Write a sentence with your answer, noting that the units on  $y_1$  and  $y_2$  are feet.