

## Homework Section 10.1.

I. In this exercise, you will make connections between systems of linear equations, and the graphs of their solutions. When you graph, use the horizontal axis for  $x_1$  and the vertical axis for  $x_2$ .

A. Read Section 1.1 to review lines and linear functions.

B. Graph the equation  $x_1 + 2x_2 = 4$ . Make sure the  $x_1$ - and  $x_2$ -intercepts are correct.

C. A system with a unique solution

i. Find a linear equation  $a_1x_1 + a_2x_2 = b$  so that the system

$$\begin{aligned}x_1 + 2x_2 &= 4 \\ a_1x_1 + a_2x_2 &= b\end{aligned}$$

has exactly one solution.

ii. Add the graph of  $a_1x_1 + a_2x_2 = b$  to your graph from part (B) using a different color.

iii. Use Gauss-Jordan Elimination to solve the system. Make sure your solution coincides with what is shown in the graph.

D. A system with no solution

i. Find a linear equation  $c_1x_1 + c_2x_2 = d$  so that the system

$$\begin{aligned}x_1 + 2x_2 &= 4 \\ c_1x_1 + c_2x_2 &= d\end{aligned}$$

has no solutions.

ii. Add the graph of  $c_1x_1 + c_2x_2 = d$  to your graph from part (B), using yet another color.

iii. Use Gauss-Jordan Elimination to solve the system. Write a sentence saying how you can tell from the resulting matrix that the system has no solutions.

E. A system with infinitely many solutions

i. Find a linear equation  $f_1x_1 + f_2x_2 = g$  so that the system

$$\begin{aligned}x_1 + 2x_2 &= 4 \\ f_1x_1 + f_2x_2 &= g\end{aligned}$$

has an infinite number of solutions.

ii. Add the graph of  $f_1x_1 + f_2x_2 = g$  to your graph from part (B) using yet another color.

iii. Use Gauss-Jordan Elimination to solve the system. Write the general solution for this system. (See Examples 4 and 5 pages 544-545.)

II. In this exercise, you will consider how the graphs of the linear equations represented in a system change as we use the third row operation (see the blue box on page 536).

A. Graph the two lines in the system of equations

$$\begin{aligned}x_1 - 2x_2 &= 0 \\ 2x_1 + 2x_2 &= 6\end{aligned}$$

Label your graph, “Graph of the lines  $x_1 - 2x_2 = 0$  and  $2x_1 + 2x_2 = 6$ .” Indicate where the solution set to this system of equations lies on the graph.

- B. Write the augmented matrix corresponding to this system (label it “Augmented matrix for the system  $x_1 - 2x_2 \dots$ ”).
- C. Perform the row operation  $(-2)R_1 + (1)R_2 \rightarrow R_2$  to produce a new matrix (label it “Augmented matrix after performing  $(-2)R_1 + (1)R_2 \rightarrow R_2$ ”).
- D. Write the system of equations corresponding to this augmented matrix (label it, and continue labeling your answers to each of the following steps appropriately).
- E. Graph the two lines in this new system of equations on a new coordinate axes. Indicate where the solution set to this system of equations lies on the graph.
- F. Obtain a new augmented matrix by applying the row operation  $6R_1 + 2R_2 \rightarrow R_1$ .
- G. Write the corresponding system of equations.
- H. Graph the two lines in this system on a new coordinate axes. Indicate where the solution set lies on the graph.
- I. Do the last steps of row reduction (continuing to label everything),  $(1/6)R_1 \rightarrow R_1$  and  $(1/6)R_2 \rightarrow R_2$ , and use the resulting augmented matrix to reveal the coordinates of the solution.
- J. Write “Checked!” if your answer corresponds to the solution shown on your graphs.