Drill Quiz Number 1 for Chapter 12 Section 4 and Chapter 13 Section 1

1. A coin is flipped and a die is rolled (both fair). The sample space for this experiment consists of pairs in which the first entry is a head or a tail, and the second entry is a number from 1 to 6. Let X be the random variable given as follows. If the coin flip is heads, then X is 1 for an even die roll and 2 for an odd die roll. If the coin flip is tails, then X is 1 for a die roll that is 2 or higher, and X is 3 for a die roll of 1.

a. (1 point) Fill out this table of values for X.

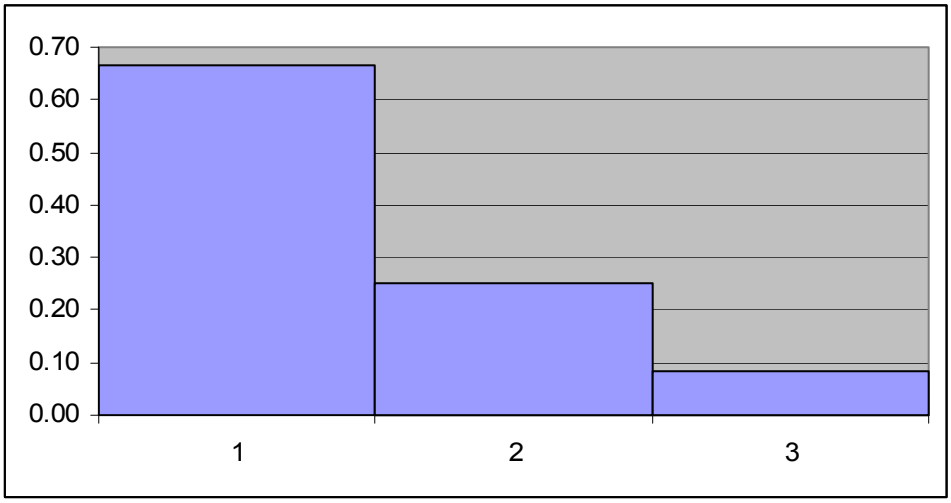
outcome	H-1	H-2	H-3	H-4	H-5	H-6
x	2	1	2	1	2	1

outcome	T-1	T-2	T-3	T-4	T-5	T-6
x	3	1	1	1	1	1

b. (3 points) Write a probability distribution for X.

x	1	2	3
P(X=x)	8/12	3/12	1/12

c. (2 points) Draw a histogram for X on the back of this quiz.



d. (1 points) Calculate P(X≤2) and shade the region in the histogram whose area is equal to that probability.

$P(X\leq 2)=8/12+3/12=11/12$. The bars above 1 and 2 should be shaded.

e. (3 points) Calculate the expected value of X.

$E(X)=1*(8/12)+2*(3/12)+3*(1/12)=17/12=1.42$

2. (2 points) State the definition of a probability density function.

See the blue box entitled Probability Density Function on page 731.

3. (3 points) Decide whether or not the function $f(x) = \frac{4x^3}{81}$ is a probability density function on the interval [0,3].

We must check to see if this function satisfies (1) and (2) in the blue box cited above. For (1), we must check that f(x) is greater than or equal to zero on the given interval. When you cube a number that is greater than or equal to zero, it remains greater than or equal to zero; similarly, when you divide a number that is greater than or equal to zero by 81 and multiply it by 4, it remains greater than or equal to zero. So since all of the numbers in our interval are greater than or equal to zero, our function satisfies item (1).

For (2), we must check that $\int_0^3 \frac{4x^3}{81} dx = 1$. We integrate, using the power rule.

$$\int_0^3 \frac{4x^3}{81} dx = \left(\frac{4}{81}\right)\left(\frac{1}{4}\right)x^4\Big|_0^3 = \left(\frac{4}{81}\right)\left(\frac{1}{4}\right)3^4 - \left(\frac{1}{81}\right)\left(\frac{1}{4}\right)0^4 = 1.$$

Thus $f(x)$ is a probability density function.