Calculus for Biology Newberger
Spring 2007

Drill Quiz for Chapter 10 Section 4

1. Suppose A and B are nxn matrices. What does it mean for A and B to be inverses of each other?
   Square matrices A and B are inverses of each other if AB=I and BA=I, where I is the identity matrix.

2. Determine whether or not the given matrices are inverses of each other. Explain your answer.
   a. $A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

   $AB = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $BA = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, so A and B are inverses of each other.

   b. $A = \begin{bmatrix} -1 & 0 & 3 \\ 0 & 0 & 2 \\ 4 & 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 & 6 \\ -10 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

   $AB = \begin{bmatrix} -1 & 0 & 3 \\ 0 & 0 & 2 \\ 4 & 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 & 6 \\ -10 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 4 \\ -2 & 0 & 35 \end{bmatrix}$, which is not the 3x3 identity matrix, so A and B are not inverses of each other.

3. On the back of the quiz, find the inverse of the matrix A. Write your answer here, and check that it is correct by multiplying it by the given matrix.

$$A = \begin{bmatrix} -1 & -1 & -1 \\ 4 & 5 & 0 \\ 0 & 1 & -3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} -1 & -1 & -1 & 1 & 0 & 0 \\ 4 & 5 & 0 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-1R1 \to R1} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & -1 & 0 & 0 \\ 4 & 5 & 0 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-4R1+R2 \to R2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & -4 & 4 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-1R2+R1 \to R1} \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & -5 & -1 & 0 \\ 0 & 1 & -4 & 4 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-1R2+R3 \to R3} \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & -5 & -1 & 0 \\ 0 & 1 & -4 & 4 & 1 & 0 \\ 0 & 0 & 1 & -4 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{-5R3w+R1 \to R1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 15 & 4 & -5 \\ 0 & 1 & -4 & 4 & 1 & 0 \\ 0 & 0 & 1 & -4 & -1 & 1 \end{array}\right] \xrightarrow{4R3w+R2 \to R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 15 & 4 & -5 \\ 0 & 1 & 0 & -12 & -3 & 4 \\ 0 & 0 & 1 & -4 & -1 & 1 \end{array}\right]$$

$A=\begin{bmatrix}-1 & -1 & -1\\ 4 & 5 & 0\\ 0 & 1 & -3\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}15 & 4 & -5\\ -12 & -3 & 4\\ -4 & -1 & 1\end{bmatrix}$. Check that

$$AA^{-1}=\begin{bmatrix}-1 & -1 & -1\\ 4 & 5 & 0\\ 0 & 1 & -3\end{bmatrix}\begin{bmatrix}15 & 4 & -5\\ -12 & -3 & 4\\ -4 & -1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$ and

$$A^{-1}A=\begin{bmatrix}15 & 4 & -5\\ -12 & -3 & 4\\ -4 & -1 & 1\end{bmatrix}\begin{bmatrix}-1 & -1 & -1\\ 4 & 5 & 0\\ 0 & 1 & -3\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$