Exam 1 Review Sheet

This exam covers chapter 10.

The following list describes the content of the exam. Practice these techniques off of the Drill problems and quizzes, in addition to problems listed below.

Section 10.1
You *will* be asked to

- Solve a system of equations using Gauss-Jordan elimination. There are three possible results: the system will either have a unique solution, no solution or infinitely many solutions. In the case that a system has infinitely many solutions, you will be asked to write the general solution for that system. When the solution does not exist, you will be asked to explain how you can tell from the augmented matrix that this is the case.

You *may* be asked to

- Graph each equation in a system with equations and two unknowns (so the graphs will be lines) that has a unique solution, infinitely many solutions and/or no solution. When there is one, you many be asked to label the solution on the graph.

Section 10.3
You *will* be asked to

- Determine when given matrices can be multiplied, and multiply them if possible.

You *may* be asked to

- Answer questions about the dimensions of matrices when they can or cannot be multiplied.

Section 10.4
You *will* be asked to

- Find the inverse of a given matrix.

You *may* be asked to

- Determine whether or not a given matrix has an inverse, and find the inverse if it exists. You may, if you like, use the determinant of the matrix to determine whether or not it has an inverse. (If the determinant is 0 then the inverse does not exist; if the determinant is non-zero, then the inverse does exist.)
- Determine if two given matrices are inverses of each other.
- Write a system of equations as a matrix equation. (See Example 3 page 577).

- Use the inverse of a matrix to solve a system of equations.
- Explain what it means for matrices A and B to be inverses of each other.

Section 10.5

You *will* be asked to

- Calculate the eigenvalues and eigenvectors for a given matrix. This may be part of solving a problem involving a Leslie matrix for a population. See problems 14-19 page 594.

You *may* be asked to

- Calculate the determinant of a given matrix.
- Use a Leslie matrix for a population to calculate the population in a given year. (For example, you may be given the population in year 1 and asked to find it in year 2.)
- Answer questions (maybe True/False?) that show you understand the following equivalences:

Let M be a 2x2 matrix.

Case 1. If the determinant of *M* is not equal to zero, then the following statements about *M* hold.

a. The matrix *M* has an inverse.

b. The system $M\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ has a unique solution, for any vector $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.

c. The system $MX = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has a unique solution, namely $X = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Case 2. If the determinant of M is equal to zero, then the following statements about M hold.

a. The matrix M does not have an inverse.

b. The system $M\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ either has no solution or infinitely many solutions, depending on the numbers in $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. (In this case, you may be asked to give an example of a vector *B* for which the system has no solution or infinitely many solutions.)

c. The system $MX = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has infinitely many solution, including $X = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Sometimes we say $MX = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has nonzero solutions, in this case.