Topology I, Newberger, Spring 2005

Homework: Sections 23 and 24. Due Tuesday, March 29th.

Follow the instructions carefully. Write your answer so that I do not have to look up the problems in the book *or on the assignment* in order to understand your responses. It is sufficient but not necessary for you to copy the problems onto your homework to achieve this.

I. Memorize the definition of connected, and learn the proofs of Theorems 23.3, 23.4, 23.5 and 23.6.

II. (10 points) Read problem #1 page 152. Suppose $\mathcal{T}' \supset \mathcal{T}$. Answer each of these two questions with a proof or a counterexample: If X is connected with respect to $\mathcal{T}$, then is it necessarily connected with respect to $\mathcal{T}'$? If X is connected with respect to $\mathcal{T}$, then is it necessarily connected with respect to $\mathcal{T}'$?

III. (10 points) Do problem #3 page 152. Begin by supposing that $A \cup \bigcup A_\alpha$ is not connected.

IV. (10 points) Choose one of the following:
   - A. Fundamental: do both of the following.
     - i. Do problem #7 page 152.
     - ii. Do problem #4 page 152.
   - B. Challenging: do problem #1(a) and (c) page 157.

V. (10 points) Do problem #10 page 158, using this outline:
   - a. Let $S = \{x \in U |$ there exists a path in U connecting x to $x_o\}$. Show S is nonempty.
   - b. Prove that S is open. Begin by letting $y \in S$. Find a radius $r > 0$ such that $B(y, r) \subset U$, and then prove that $B(y, r) \subset S$. Explain why this shows S is open.
   - c. Prove that S is closed. Begin by letting $x_n \to x$ in U, with $x_n \in S$. Prove that x is in S. Explain why this implies that X is closed (See the sequence lemma page 130).
   - d. Show $S = U$ by showing that $U - S$ is empty. Use contradiction: suppose $U - S$ is nonempty and show that S and $U - S$ form a separation of U, contradicting the connectedness of U.