Topology I, Newberger, Spring 2005

Homework: Sections 20 and 21. Due Thursday, March 15th.

Follow the instructions carefully. Write your answer so that I do not have to look up the problems in the book *or on the assignment* in order to understand your responses. It is sufficient but not necessary for you to copy the problems onto your homework to achieve this.

I. (10 points)
   A. Prove that the function d' given in problem #1.a. on page 126 is a metric. Sketch typical basis elements for the topology induced by this metric in $\mathbb{R}^2$.
   B. Do *one* of the following:
      - Fundamental: Prove the metric in #1.a. induces the product topology on $\mathbb{R}^n$.
      - Challenging: Prove the metric in #1.b. induces the product topology on $\mathbb{R}^n$.

II. Problem #2 on page 126 says that the dictionary topology on $\mathbb{R}\times\mathbb{R}$ is metrizable. Figure out what the metric is and convince yourself that it is indeed a metric. Do not turn in your work.

III. A. Read the definition of convergence on page 98.
   B. (10 points) Let X and Y be topological space, and consider $X \times Y$ with the product topology. Let (x_n) be a sequence of elements of X and let (y_n) be a sequence of points in Y; let $x \in X$ and $y \in Y$. Prove that if (x_n) converges to x and (y_n) converges to y then $x_n \times y_n$ converges to $x \times y$ in the product topology on $X \times Y$. The converse of this statement also holds, but you need not prove it here.
   C. (10 points)
      i. Do problem #12 parts (a) and (b) on page 135. Note that each part of this problem is asking you to show that a certain function $f : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ is continuous with respect to the topology on $\mathbb{R} \times \mathbb{R}$ induced by ρ and the topology on $\mathbb{R}$ induced by d. In part (a), the function in question is $f(x, y) = x + y$; in part (b), the function is $f(x, y) = xy$.
      ii. Let $x_n \to x$ and $y_n \to y$ in $\mathbb{R}$. Use the results of (a) and (b) to prove that $x_n + y_n \to x + y$ and that $x_n y_n \to xy$.

IV. (10 points) Do problem #8 on page 134.