Topology I, Newberger, Spring 2005

Homework: Section 17 Tips and Remarks.

III. #16(a) page 101. Let $K = \{1/n | n \in \mathbb{Z}_+\}$.
$\mathcal{T}_1$ = the standard topology. Prove that $\bar{K} = K \cup \{0\}$. We did this in class.

$\mathcal{T}_2$ = the topology of $\mathbb{R}_K$. Prove that K is closed in this topology, so $\bar{K} = K$. Prove that K is closed by proving that its complement, $\mathbb{R} - K$, is open in this topology. Prove that $\mathbb{R} - K$ is open by writing it as a union of basis elements.

$\mathcal{T}_3$ = the finite complement topology. Prove that $\bar{K} = \mathbb{R}$. Show that the only closed set containing K is $\mathbb{R}$. Explain that this implies that the closure of K is $\mathbb{R}$, since the closure is the intersection of all closed sets containing K.

$\mathcal{T}_4$ = the upperlimit topology. Prove that $\bar{K} = K$. Prove that K is closed in this topology by showing that its complement is open. Do this by writing $\mathbb{R} - K$ as a union of basis elements for $\mathcal{T}_4$.

$\mathcal{T}_5$ = the topology generated by sets of the form $(-\infty, a)$. Prove that $\bar{K} = [0, \infty)$. Show this in two steps: First show that for each $x \in (-\infty, 0)$, there is an open set for this topology that contains x but does not intersect K. Second show that if $x \in [0, \infty)$, then any open set containing x intersects K.

IV. #18. Do not redo this whole problem. Prove only that

$$\bar{D} = D \cup ([0, 1) \times \{1\}) \cup ((0, 1] \times \{0\}).$$

First show that any neighborhood of a point in the set given here must intersect D. Then show that any point in the complement of the set given here has a neighborhood that does not intersect D (consider 0×0 and 1×1 as separate cases).