

## Topology I, Newberger, Spring 2005

**Homework: Sections 14,15,16.** Due Tuesday, February 15th.

Follow the instructions carefully. Write your answer so that I do not have to look up the problems in the book *or on the assignment* in order to understand your responses. It is sufficient but not necessary for you to copy the problems onto your homework to achieve this.

- I. (10 points)
  - A. Read the definition of a projection mapping on page 87, as well as Theorem 15.2 on page 88.
  - B. Let  $X$  and  $Y$  be topological spaces, and consider  $X \times Y$  as a topological space with the product topology. Assume that  $U$  is an open set in  $X \times Y$ . What do you know about  $U$ ?
  - C. Do problem #4 on page 92.
- II. (10 points) Do problem #3 on page 92.
- III. Consider  $\mathbb{R}^2$  with the product topologies  $\mathbb{R}_l \times \mathbb{R}$  and  $\mathbb{R}_l \times \mathbb{R}_l$ .
  - A. (10 points) Explicitly write down bases for the product topologies  $\mathbb{R}_l \times \mathbb{R}$  and  $\mathbb{R}_l \times \mathbb{R}_l$ . Are the following sets open in either  $\mathbb{R}_l \times \mathbb{R}$  or  $\mathbb{R}_l \times \mathbb{R}_l$ ? Prove your answers are correct.
    - i.  $\{(x, y) | x \geq 1\}$ .
    - ii.  $\{(x, y) | x \leq 1\}$ .
    - iii.  $\{(x, y) | y \geq x\}$
  - B. (10 points) Do problem #8 on page 92. Hint: not all lines are the same.
- IV.
  - A. Reread Lemma 13.3.
  - B. (10 points each) Do problems #9 and #10 on page 92. “Compare two topologies” means write a proof showing that the topologies are equal, or that one is strictly finer than the other, or that they are not comparable.