Topology I, Newberger, Spring 2005

Homework: Sections 7, 12 and 13. Due Tuesday, February 8th.

Follow the instructions carefully. Write your answer so that I do not have to look up the problems in the book in order to understand your responses. It is sufficient but not necessary for you to copy the problems onto your homework to achieve this.

I. Memorize the definition of the words topolological space, comparable, finer, coarser, basis, and topology generated by a basis. Understand the statements of Lemmas 13.1, 13.2 and 13.3.

II. Read Section 7.

III. (10 points)
   A. Suppose A is countable. Prove that if there is a surjection from A to a set B, then B is countable. Prove that if there is an injection from a set B into A, then B is countable. These proofs should be short.
   B. Do 5 parts of problem #5 on page 51 (you choose).

IV. (10 points) Do problem #3 on page 83.

V. (10 points)
   A. Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be topologies on a set X. Write a sentence saying what it means for $\mathcal{T}_1$ to be comparable to $\mathcal{T}_2$. Write a sentence saying what you must do to show that $\mathcal{T}_1$ and $\mathcal{T}_2$ are not comparable (be specific).
   B. Read the definition starting at the bottom of page 81 and continuing on page 82.
   C. Do problems #6 on page 83.

VI. (10 points) Do problem #7 on page 83. In this problem you are asked to discuss the comparability of various topologies on $\mathbb{R}$. Provide either a proof or a counterexample for each pair of topologies. Since this is a lot of arguments, look for ways to make your proofs concise.