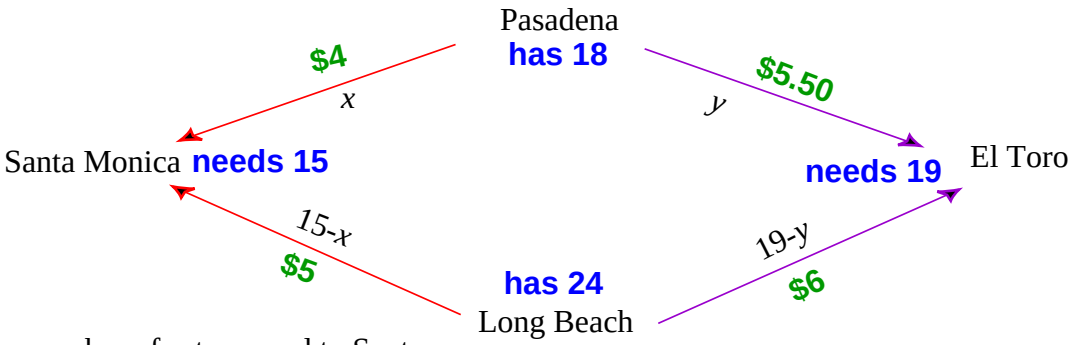
Newberger, College Algebra 112-10, Spring 2005

Worksheet: Linear Programming Problems.

Problem 9, page 509.

An electronics discount chain has a sale on a certain brand of sterio. The chain has stores in Santa Monica and El Toro and warehouses in Long Beach and Pasadena. To satisfy rush orders, 15 sets must be shipped from warehouses to the Santa Monica store, and 19 must be shipped to the El Toro store. The cost of shipping a set is \$5 from Long Beach to Santa Monica, \$6 from Long Beach to El Toro, \$4 from Pasadena to Santa Monica, and \$5.50 form Pasadena to El Toro. The Long Beach warehouse has 24 sets and the Pasadena warehouse has 18 sets in stock. How many sets should be shipped from each warehouse to each store to fill the orders at a minimum shipping cost?

1. Use the diagram below to help you organize the information given in the problem. Write the information on the diagram.



The number of sets moved to Santa Monica (along the red arrows) must add up to 15. I am going to choose x to be the number of sterios moved from Pasadena to Santa Monica. In that case, the number of sterios moved from Long Beach to Santa Monica will be 15-x. The solution I am writing will be based on this choice of x.

There is noting wrong with choosing x to be the number of sterios moved from Long Beach to Santa Monica; in that case 15-x sterios will be moved from Pasadena to Santa Monica.

The number of sets moved to El Toro (along the purple arrows) nust add up to 19. I am going to choose y to be the number of sterios from Pasadena to El Toro. The the number of sterios moved from Long Beach to El Toro will be 19-y. the solution I am writing will be based on this choice of y.

There is nothing wrong with choosing y to be the number of sterios moved from Long Beach to El Toro; in that case, the number of sterios moved from Pasadena to El Toro will be 19-y.

2. The question asks for each of the following pieces of information. Two of these will be your variables x and y. You can express the other two in terms of x and y. Write the variables or formulas that describe each item on the list below.

Verbal description	Variable or formula
Number of sets shipped from Pasadena to Santa Monica	x
Number of sets shipped from Long Beach to Santa Monica	$15 - x$
Number of sets shipped from Pasadena to El Toro	y
Number of sets shipped from Long Beach to El Toro	$19 - y$

3. Write a formula for the objective function $C(x, y)$, and simplify it.

$$C(x, y) = 4(x) + 5(15 - x) + 5.5(y) + 6(19 - y)$$

Simplify this:

$$\begin{aligned} C(x, y) &= 4(x) + 5(15 - x) + 5.5y + 6(19 - y) \\ &= 4x + 75 - 5x + 5.5y + 114 - 6y \\ &= -x - 0.5y + 189 \end{aligned}$$

4. Write the formulas for constraints (inequalities) that you can get from this problem. Label each with a verbal reason for that constraint.

Verbal description	inequalities
The numbers of sets shipped from each location are all greater or equal to zero	$x \geq 0$ $15 - x \geq 0$ $y \geq 0$ $19 - y \geq 0$
The number of sets leaving Long Beach is $(15 - x) + (19 - y)$. This must be less than or equal to the number of sets available in Long Beach, which is 24.	$(15 - x) + (19 - y) \leq 24$
The number of sets leaving Pasadena is $x + y$. This must be less than or equal to the number of sets available in Pasadena, which is 18.	$x + y \leq 18$

5. Graph the feasible region. Graph the inequalities that only have one variable first.

- **Inequality:** $x \geq 0$.
  Line: The line associated to this inequality is $x = 0$. Lines of the form $x = a$ are vertical. The line $x = 0$ is the y-axis.
  Check a point that does not lie on the line: (Here, our favorite point, $(0, 0)$, is on the line, so we should use a different point.) Check $(x, y) = (1, 2)$. We get $1 \geq 0$, which is true, so we will shade the side of the line containing the point $(1, 2)$. (i.e. shade the right side of the line.)

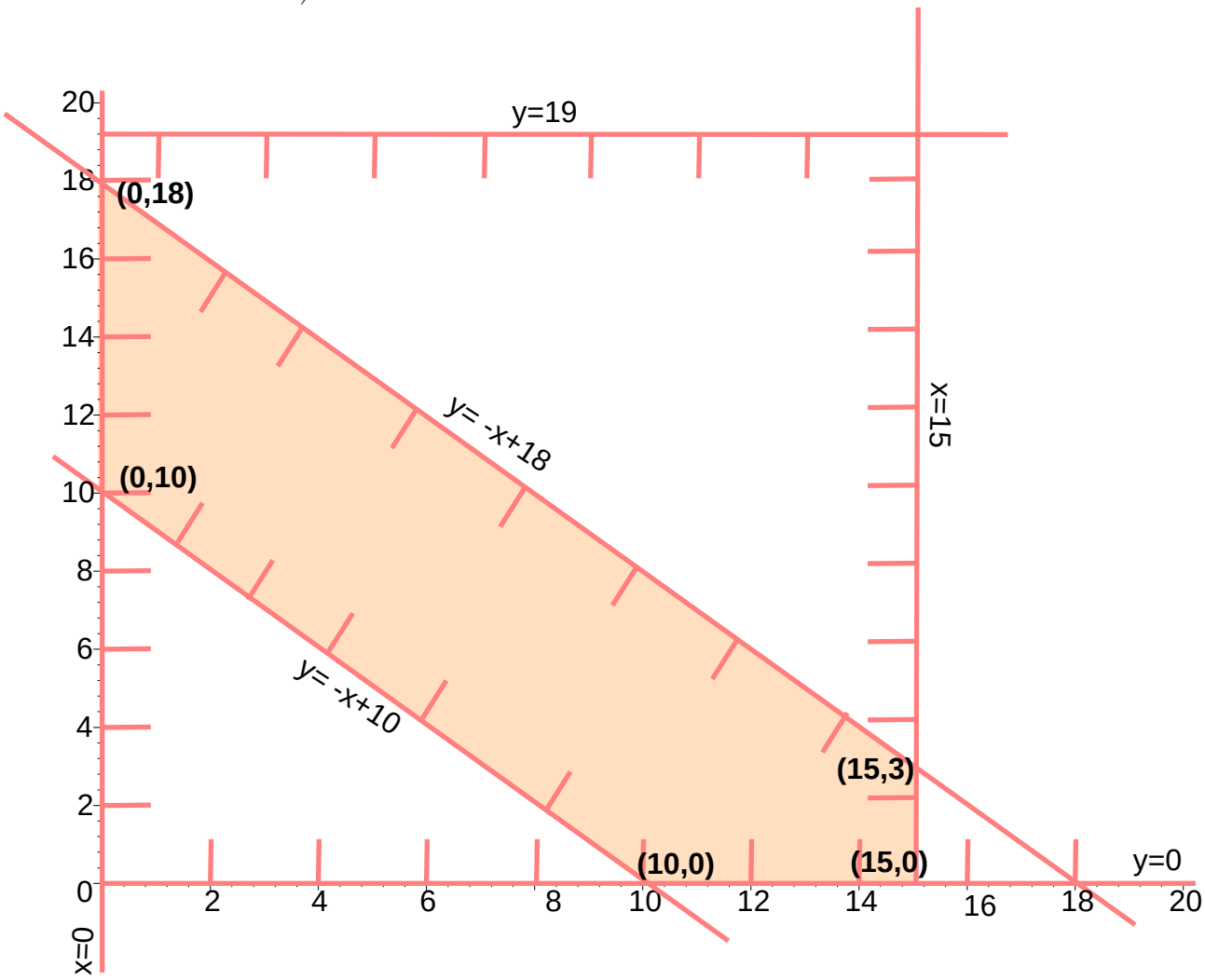
- **Inequality:** $y \geq 0$.
  Line: The line associated to this inequality is $y = 0$. Lines of the form $y = a$ are horizontal. The line $y = 0$ is the x-axis.
  Check a point that does not lie on the line: (Here, our favorite point, $(0, 0)$, is on the line, so we should use a different point.) Check $(x, y) = (1, 2)$. We get $2 \geq 0$, which is true, so we will shade the side of the line containing the point $(1, 2)$. (i.e. shade above the line.)

- **Inequality:** $15 - x \geq 0$.
  Line: The line associated to this inequality is $15 - x = 0$. Solve for x: $x = 15$. Lines of the form $x = a$ are vertical. This is the vertical line that crosses the x axis at 15.
  Check a point that does not lie on the line: (Here, our favorite point, $(0, 0)$, is on the not on the line, so we can use it.) Check $(x, y) = (0, 0)$. We get $15 - 0 \geq 0$, which is true, so we will shade the side of the line containing the point $(0, 0)$. (i.e. shade the left side of the line.)

- **Inequality:** $19 - y \geq 0$.
  Line: The line associated to this inequality is $19 - y = 0$. Solve for y: $y = 19$. Lines of the form $y = a$ are horizontal. This is the horizontal line that crosses the y axis at 19.
  Check a point that does not lie on the line: (Here, our favorite point, $(0, 0)$, is on the not on the line, so we can use it.) Check $(x, y) = (0, 0)$. We get $19 - 0 \geq 0$, which is true, so we will shade the side of the line containing the point $(0, 0)$. (i.e. shade below the line.)

- **Inequality:** $(15 - x) + (19 - y) \leq 24$.
  Line: The line associated to this inequality is $(15 - x) + (19 - y) = 24$. Solve for y and write it in slope-intercept form: $y = -x + 10$. This line has slope -1 and intercept 10.
  To graph it, we will find the x-intercept (by setting $y = 0$ and solving for x; i.e. we solve $0 = -x + 10$ for x, and get $x = 10$. Plot the y-intercept (0,10), and the x-intercept $(10, 0)$ and connect them with a line.
  Check a point that does not lie on the line: (Here, our favorite point, $(0, 0)$, is on the not on the line, so we can use it.) Check $(x, y) = (0, 0)$. We get $(15-0)+19-0) \leq 24$, which is false, so we will shade the side of the line that does not contain the point $(0, 0)$. (i.e. shade above the line.)

- **Inequality:** $x + y \leq 18$.
  Line: The line associated to this inequality is $x + y = 18$. Solve for y and write it in slope-intercept form: $y = -x + 18$. This line has slope -1 and intercept 18. Since the slope is the same as the slope of the line above, this line will be parallel to that line.
  To graph it, we will find the x-intercept (by setting $y = 0$ and solving for x; i.e. we solve $0 = -x + 18$ for x, and get $x = 18$. Plot the y-intercept (0,18), and the x-intercept $(18, 0)$ and connect them with a line.
  Check a point that does not lie on the line: (Here, our favorite point, $(0, 0)$, is on the not on the line, so we can use it.) Check $(x, y) = (0, 0)$. We get $0 + 0 \leq 18$, which is true, so we will shade the side of the line containing the point $(0, 0)$. (i.e. shade below the line.)



6. Find the coordinates for the vertices of the feasible region.

   We can see on the graph which vertices we need to calculate.
   - The line $x = 0$ crosses the line $y = -x + 10$. Find the intersection by substituting $x = 0$ into $y = -x + 10$. We get $y = 10$. So the coordinates are $(0, 10)$. (You could have also done this one by looking at the picture, since this vertex is on an axis).
   - The line $x = 0$ crosses the line $y = -x + 18$. Find the intersection by substituting $x = 0$ into $y = -x + 18$. We get $y = 18$. So the coordinates are $(0, 18)$. (You could have also done this one by looking at the picture, since this vertex is on an axis).
   - The line $x = 15$ crosses the line $y = -x + 18$. Find the intersection by substituting $x = 15$ into $y = -x + 10$. We get $y = 3$. So the coordinates are $(15, 3)$.
   - The line $y = 0$ crosses the line $x = 15$. So the coordinates are $(0, 15)$. (You could have also done this one by looking at the picture, since this vertex is on an axis).
   - The line $y = 0$ crosses the line $y = -x + 10$. Find the intersection by substituting $y = 0$ into $y = -x + 10$. We get $x = 10$. So the coordinates are $(10, 0)$. (You could have also done this one by looking at the picture, since this vertex is on an axis).

7. Evaluate the objective function at each of the vertices and store the information here.

Vertex (x, y)	Objective function $C(x, y) = -x - 0.5y + 189$
(0,10)	$C(0, 10) = 184$
(0,18)	$C(0, 18) = 180$
(15,3)	$C(15, 3) = 172.5$
(0,15)	$C(0, 15) = 181.50$
(10,0)	$C(10, 0) = 179$

8. Identify the coordinates (x, y) at which the objective function is minimized on the table above.

   The minimum cost happens when $(x, y) = (15, 3)$.

9. How many sets the should be shipped from each warehouse to each store to fill the orders at a minimum shipping cost?

   I included this table, because I thought it would help you answer the question above. In the second column, I just put in the entries from the original table in part 2. In the third column, I put in $x = 15$ and $y = 3$.

Verbal description	Variable or formula	Value to minimize cost
Number shipped from Pasadena to Santa Monica	x	15
Number shipped from Long Beach to Santa Monica	$15 - x$	0
Number shipped from Pasadena to El Toro	y	3
Number of sets shipped from Long Beach to El Toro	$19 - y$	16

So, finally (!), we get that to minimize the shipping cost, we will move 15 sets from Pasadena to Santa Monica, 0 sets from Long Beach to Santa Monica, 3 sets from Pasadena to El Toro and 16 sets from Long beach to El Toro. The cost they will have to pay if they do this is \$172.50.