

## Computer Project 3: Transforming Graphs

**Due:** Tuesday, April 11. Note, you have two weeks for this lab; do it early, so you can get your questions answered and turn it in on time!

**Goals:** The goal of this project is to understand the relationship between changes in a function's formula and changes in its graph.

- Open Microsoft Excel. In the file menu, choose Page setup... Click on Landscape, and click OK.

### I. Shifting the graph up and down.

- Type "Shifting the graph up and down" in cell A1. Type "x" in cell A2 and " $f(x)=x^2$ " in cell B2.
- Type -10 into cell A3. In Cell A4, type  $=A3+1$  (including the equals sign), and hit enter or click check in the formula bar. Here is how to fill this formula down to cell A23. With cell A4 selected, put your cursor on the bottom right hand side of the cell. Your cursor should change into a small black plus shape. Hold the mouse so that the cursor shows that plus shape, press the left mouse button, and holding it down, drag to the left to cell A23. You should now see the numbers -10 through 10 in these cells.
- In cell B2, type " $f(x)=x^2$ ". In cell B3, type the formula  $=A3^2$ . Fill this formula down to cell B23, as you did above. Now in columns A and B, you have a table of values for the function  $f(x)=x^2$ .
- In cell C2, type " $g(x)=x^2+50$ ." In cell C3, type the formula  $=A3^2 + 50$ . Fill this formula down to cell C23.
- In cell D2, type " $h(x)=x^2-100$ ." In cell D3, type the formula  $=A3^2 - 100$ . Fill this formula down to cell D23.
- Save your work.

You will now graph these three functions.

- Select cells A2 through D23. Note that you are selecting the column headings as well as the data.
- Choose Chart... from the Insert menu. A dialog box called "Chart Wizard – Step 1 of 4 – Chart Type" will appear. Click on x-y scatter plot. There are five chart subtypes shown. Click on the second one (with the points and the curve showing). Click Next.
- The title of the dialog box changes to Step 2 of 4 – Chart Source Data. We won't use these options at this time. Click Next.
- Step 3 of 4 – Chart Options. Click on the Titles tab, if it is not already selected. Type "Shifting the graph up and down" for the title of your chart. Click the tab at the top of the dialog box that says Gridlines. Check Major gridlines under Value (X) axis and Value (Y) axis. Click Next.
- Step 4 of 4 – Chart Location. This dialog box allows you to choose whether the chart will appear in a new window, or as a part of the worksheet containing the data. Mark the "As an object in" radio button, if it is not already marked. Click Finish.

Formatting:

- Make sure the chart is not selected by clicking on cell A1. To make the page breaks visible, choose Print Preview... from the File menu. Click close.
- Click on Cell A25, and choose Page Break from the Insert menu.
- Now, click on the white background of the chart and drag the chart onto the page next to the data. You may need to adjust the chart's size to make it fit on the page. Do this by clicking on the chart to activate it, and then clicking and dragging one of the corners of the chart, to adjust its size.

**Questions** (You can do these after you have done all of the excel work, but skim over them to make sure you understand what they mean now, so that you can ask if you have any questions.):

1. In this assignment, I am going to ask you about the relationships between graphs and between formulas. Here is an example, showing what I mean by “relationship.”

Here is the relationship between the formulas for  $f(x)$  and  $g(x)$ :

$g(x)$  is equal to  $f(x)$  plus 50.

Here is the relationship between the graphs of  $f$  and  $g$ :

The graph of  $g(x)$  looks the same as the graph of  $f(x)$ , shifted up by 50 units.

Write a sentence explaining the relationship between the formula for  $f(x)$  and the formula for  $h(x)$ , and a second sentence explaining the relationships between the graph of  $f(x)$  and the graph of  $h(x)$ .

2. What is the formula for the function whose graph looks like the graph of  $f(x)$ , shifted down by 7 units?
3. Describe in words what the graph of  $p(x)=x^2-30$  looks like.
4. The coordinates of the vertex of  $f(x)$  are  $(0,0)$ . What are the coordinates of the vertex of  $g(x)$  of  $h(x)$ ?
5. The range of a function is the set of all values the function yields over its domain. You can see the range of the function on its graph by looking at the  $y$ -values. For example, the range of  $f(x)$  is  $[0,+\infty)$ . Find the range of  $g(x)$  and  $h(x)$ .
6. If the range of a function  $q(x)$  is the interval  $[2,3]$ , what is the range of the function  $q(x)+10$ ?

## II. Shifting the graph left and right.

- Type “Shifting the graph left and right” in cell A25.
- Select cells A2 through B23. Choose Copy from the edit menu, or press control-C. Click on cell A26, and choose Paste from the edit menu, or press control-V.
- In cell C26, type “ $g(x)=(x+5)^2$ .” In cell C27, type the formula  $=(A27+5)^2$ . Fill this formula down to cell C47.
- In cell D26, type “ $h(x)=(x-7)^2$ .” In cell D27, type the formula  $=(A27-7)^2$ . Fill this formula down to cell D47.
- Select cells A26 through D47, and choose Chart... from the Insert menu. Follow the instructions in Part I to create a chart showing the graphs of these three functions.
- Click on Cell A50, and choose Page Break from the Insert menu. Adjust the size and location of the chart so that it lands on the page with the data.

### Questions:

1. Here is the relationship between the formulas for  $f(x)$  and  $g(x)$ :  
 $g(x)$  is equal to  $f(x+5)$ .

Here is another way of saying the same thing:

To get the formula for  $g(x)$ , replace the  $x$  in the formula for  $f(x)$  by  $x+5$ . (The formula for  $f(x)$  is  $x^2$ , so the formula for  $f(x+5)$  is  $(x+5)^2$ ; the  $x$  in the formula for  $f(x)$  was replaced by  $x+5$  to get the formula for  $g(x)$ .)

Write a sentence explaining the relationship between the formula for  $f(x)$  and the formula for  $h(x)$ .

2. Write a sentence explaining the relationships between the graph of  $f(x)$  and the graph of  $g(x)$ . Do the same for  $h(x)$ .
3. What is the formula for the function whose graph looks like the graph of  $f(x)$ , shifted to the right by 7 units?
4. Describe in words what the graph of  $p(x)=(x-30)^2$  looks like.
5. What are the coordinates of the vertex of  $g(x)$ ? of  $h(x)$ ?
6. Find the range of  $g(x)$  and  $h(x)$ .
7. If the range of a function  $q(x)$  is the interval  $[2,3]$ , what is the range of the function  $q(x+10)$ ?
8. The domain of a function is the set of values at which the function is defined. Here, all three functions have domain  $(-\infty, +\infty)$ , which means these functions are defined on all real numbers. Say instead you had a function  $b(x)$  whose domain is the interval  $[-1,3]$ . What is the domain of the function  $b(x-5)$ ?

### III. Reflecting the graph across the x-axis and the y-axis

Type “Reflecting the graph across the x-axis and the y-axis” in cell A50.

- Select cells A2 through A23. Choose Copy from the edit menu, or press control-C. Click on cell A51, and choose Paste from the edit menu, or press control-V.
- In cell B51, type “ $f(x)=x^2 + 10*x + 25$ .” In cell B52, type the formula  $=A52^2+10*A52+25$ . Fill this formula down to cell B72.
- In cell C51, type “ $g(x)=(-x)^2+10*(-x)+25$ .” Note the “-” signs. In cell C52, type the corresponding formula,  $=(-A52)^2+10*(-A52)+25$ . Fill this formula down to cell C72.
- In cell D51, type “ $h(x)=-(x^2+10*x+25)$ ,” and in cell D52, type the formula  $=(-1)*(A52^2+10*A52+25)$ . Be careful to get the ( ) around the whole thing, so that the minus one is multiplying the whole function. Note: Excel doesn’t always understand the notation  $-( ... )$ , so we write  $(-1)*( ... )$  to make sure that Excel understands what we want. Fill this formula down to D72.
- Select cells A51 through D72, and choose Chart... from the Insert menu. Follow the instructions in Part I to create a chart showing the graphs of these three functions.
- Click on Cell A80, and choose Page Break from the Insert menu. Adjust the size and location of the chart so that it lands on the page with the data.

#### Questions:

1. Here are the relationships between the formula for  $f(x)$  and the formulas for  $g(x)$  and  $h(x)$ . To get the formula for  $g(x)$ , replace the  $x$  in the formula for  $f(x)$  by  $(-x)$ . In other words,  $g(x)=f(-x)$ . To get the formula for  $h(x)$  multiply the formula for  $f(x)$  by  $-1$ . In other words,  $h(x)= - f(x)$ .

Let  $p(x)=2x^3+6$ . Write the formulas for  $b(x)=p(-x)$ , and for  $c(x)= - p(x)$ . Make your answer easy to read (so I can tell which formula is which and what you did to get them).

2. Use the expressions, “reflect(ion) across the x-axis” and “reflect(ion) across the y-axis” to write sentences explaining the relationships between the graph of  $f(x)$  and the graph of  $g(x)$ , and between the graph of  $f(x)$  and the graph of  $h(x)$ .

3. We know that  $g(x)$  and  $h(x)$  are different functions, because we can see that their graphs are different. Now let us look at the formulas, algebraically. We want to see that  $(-x)^2+10*(-x)+25$  is not the same as  $-(x^2+10*x+25)$ . On your paper write:

$$g(x)=(-x)^2+10*(-x)+25 =$$

$$h(x)= -(x^2+10*x+25) =$$

and simplify each expression. What is the difference between these two formulas?

4. What is the formula for the function whose graph looks like the reflection of the graph of  $p(x)= 2x^3+6$  across the x-axis?

5. What is the formula for the function whose graph looks like the reflection of the graph of  $p(x)= 2x^3+6$  across the y-axis?

6. What is the range of the functions  $f(x)$ ,  $g(x)$ , and  $h(x)$  (look at their graphs)?

7. If the range of  $t(x)$  is the interval  $[2,4]$  and domain  $[-3,5]$ , what are the range and domain of the functions  $u(x)=t(-x)$  and  $u(x)= - t(x)$ ?

#### IV. You choose the transformation

Graphs that are obtained by shifting and reflecting the graph of  $f(x)=x^2$  are called parabolas. In Parts I-III, you displayed graphs that show you how to manipulate the formula of  $f(x)=x^2$  in order to get a new parabola, that is shifted up, down, left or right, or reflected across the x- or y-axis.

Make up two more examples of your own, using more than one transformation. For each example:

1. Give a verbal description of what you do to the graph of  $f(x)=x^2$  to get the graph of your new parabola (use the words shift and reflect).
2. Write down the formulas for each of your examples.
3. Write down the range and vertex of each of your examples.
4. Use excel to graph both of your functions on the same graph as  $f(x)=x^2$ . Double check that the verbal description that you gave corresponds with what you see in the graphs.

Ask for help if you need it, rather than leaving parts unanswered!

Here are some examples to help you understand what to do (see the graph below):

To get a parabola that is shifted to the left by 7, the formula is  $(x+7)^2$ . The range of this parabola is the interval  $[0,+\infty)$ . The vertex of this parabola is at  $(x,y)=(-7,0)$ .

To get a parabola that is shifted to the left by 7 and then reflected across the x-axis, the formula is  $-(x+7)^2$ . To make Excel understand this one, I will type  $=(-1)*((A82+7)^2)$ . Notice that I have  $(-1)$  and  $((A82+7)^2)$  enclosed in parentheses. The range of this parabola is the interval  $(-\infty,0]$ . The vertex of this parabola is at  $(x,y)=(-7,0)$ .

To get a parabola that is shifted to the left by 7 and then reflected across the x-axis, and then shifted up by 100, the formula will be  $-(x+7)^2+100$ . To make Excel understand, I will type  $=(-1)*((A82+7)^2)+100$ . The vertex of this new parabola is at  $(x,y)=(-7,100)$ . The range of this new parabola is  $(-\infty,100]$ .

