Newberger Math 561A Spring 04

Homework: Measurable Functions

(1) (10 points) Using properties of the σ-algebra of measurable sets.
Let $E \subseteq \mathbb{R}^n$ be a measurable set. Suppose that $f : E \to [-\infty, +\infty]$. Prove the following statements from W&Z Corollary 4.2:

  (a) If f is measurable, then the sets $\{x \in E : f(x) = +\infty\}$, $\{x \in E : f(x) = -\infty\}$, $\{x \in E : f(x) > -\infty\}$, $\{x \in E : f(x) < +\infty\}$, and $\{x \in E : f(x) = a\}$ are all measurable.

  (b) The function f is measurable if and only if $\{x \in E : f(x) = -\infty\}$ is measurable and $\{x \in E : a < f(x) < +\infty\}$ is measurable for every finite $a \in \mathbb{R}$.

(2) (10 points) Making measurable functions from other measurable functions.

  (a) Prove **W&Z Theorem 4.8** Let $E \subseteq \mathbb{R}^n$ be measurable, and let $f : E \to [-\infty, +\infty]$. If f is measurable and λ is any real number, then $f + \lambda$ and λf are measurable.

  (b) Prove this part of **W&Z Theorem 4.10**: Let $E \subseteq \mathbb{R}^n$ be measurable, and let $g : E \to [-\infty, +\infty]$. Suppose g is non-zero on E. Prove that $1/g$ is measurable.

(3) (10 points) W&M Exercise 20.23.

(4) (10 points) W&M Exercise 20.15.

(5) (10 points) W&Z page 61 #3. This problem uses Theorem 4.3 to generalize "measurable" to vector valued functions.