Newberger: Math 233, Spring 2004

Worksheet: Injections, Surjections and Bijections.

(1) (10 points) Page 137 #4. Either write out the question on each part, or express your answer as a complete sentence. Answers consisting only of a numerical expression will not receive credit.

(2) (10 points) Page 138 #9. In this problem you are asked for conditions that make something happen. Write your answers as if-then statements: "If [my condition holds], then [something happens]." Justify your answers (can you draw a picture that would clarify your explanation?). (Up to 3 bonus points for a correct proof of your statements.)

(3) (10 points) Determine whether or not each of the following functions is one-to-one, onto and/or a bijection. Prove your answers are correct.
   (a) $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ given by $f(a, c) = (c, a)$.
   (b) $g : \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ given by $g(t) = (t, t)$.
   (c) $j : \mathbb{Z} \times \mathbb{R} \to \mathbb{R}$ given by $j(m, x) = (-1)^m x$.

(4) (10 points) Let $h : \mathbb{C} - \{0\} \to S^1$ be given by $h(z) = \frac{z}{|z|}$. Remember complex numbers are of the form $a + bi$ where $a, b \in \mathbb{R}$. We draw them in $\mathbb{R}^2$ by plotting $a + bi$ at (a, b).
   (a) Draw two diagrams (large enough) one showing the domain and the other the codomain of h (labelling your answers).
   (b) Why do you think I chose the domain to be $\mathbb{C} - \{0\}$ rather than simply $\mathbb{C}$?
   (c) Calculate $h(i/2)$, $h(\cos(t) + i \sin(t))$ where t is any real number, $h(2 + 3i)$, $h(-2i)$ and $h(5)$.
   (d) Draw a graph of $\mathbb{R}^2$ showing $i/2$, 5, $2+3i$ and $-2i$ (large enough). Label the image under h of each of these points on the graph. Draw and arrow from each of these points to its image under h.
   (e) Is h onto? Prove your answer is correct.
   (f) Is h one-to-one? Prove your answer is correct.

(5) (10 points) Prove that the function $k : \mathbb{R} \to (-1, 1)$ given by $k(x) = \frac{x}{\sqrt{1+x^2}}$ is a bijection. *Tip:* If you get $x_1^2 = x_2^2$, then you can conclude that $x_1 = x_2$ or $x_1 = -x_2$. If you want to conclude that $x_1 = x_2$, you must provide additional explanation showing that $x_1 = -x_2$ does not happen.