

Worksheet: Limits.

Prove the following statements. Do not include the scratch work you do to find δ or K .

1. (10 points) Let $f(x) = -2x + 5$. Prove $\lim_{x \rightarrow 2} f(x) = 1$.
2. (10 points) Let

$$h(t) = \begin{cases} 2t - 6 & \text{if } t > 3 \\ -t + 3 & \text{if } t \leq 3. \end{cases}$$

Prove $\lim_{t \rightarrow 3} h(t) = 0$.

3. (10 points) Let $j(z) = \frac{2}{z+2} + 4$. Prove $\lim_{z \rightarrow +\infty} j(z) = 4$.
4. (10 points) Let $g(y) = y^2$. Prove that $\lim_{y \rightarrow -2} g(y) = 4$.
5. (Up to 6 points extra credit) Draw diagrams (graphs) that show important aspects of your proofs for problems (1), (2), and (3).

Tips:

- If you need a reference, you should be able to find examples like these in your favorite calculus book. Look in the table of contents for something like “The precise definition of a limit.”
- If you get confused about absolute values and inequalities, try drawing the sets you are trying to understand on a number line.
- Note that $|y - (-2)| = |y + 2|$. Also note that if a and b are positive numbers, then $-a < c < -b$ implies that $b < |c| < a$ (draw the number line with the numbers $a, -a, b, -b, c$ and $|c|$ labelled).