

Worksheet: Set Inclusions, Unions, Intersections and Cartesian products.

- (1) (10 points) Let $a, b \in \mathbb{Z}$, with $a \neq 0$. Prove that $a|b$ if and only if $b\mathbb{Z} \subseteq a\mathbb{Z}$. Recall that to prove an if and only if statement you must complete two proofs:

- (a) If $a|b$, then $b\mathbb{Z} \subseteq a\mathbb{Z}$.
(b) If $b\mathbb{Z} \subseteq a\mathbb{Z}$, then $a|b$.

- (2) (10 points) Let $a, b \in \mathbb{Z}$ with $a \neq 0$ and $b \neq 0$. Let d be the greatest common divisor of a and b . Prove that

$$\{ma + nb | m, n \in \mathbb{Z}\} = d\mathbb{Z}.$$

Recall to prove that two sets are equal, you must do two proofs:

- (a) $\{ma + nb | m, n \in \mathbb{Z}\} \subseteq d\mathbb{Z}$.
(b) $\{ma + nb | m, n \in \mathbb{Z}\} \supseteq d\mathbb{Z}$.

- (3) (10 points) Let A, B and C be sets. Theorem 2.26(e) says:

$$A - (B \cup C) = (A - B) \cap (A - C).$$

- (a) Draw a Venn Diagram with three circles, and shade in the region given by $A - (B \cup C)$. On scratch paper, also draw the diagram representing $(A - B) \cap (A - C)$, and make sure you believe that Theorem 2.26(e) is true.
(b) Prove Theorem 2.26(e).

- (4) (10 points) Let A, B and C be sets. Theorem 2.45(b) says

$$(A \cap B) \times C = (A \times C) \cap (B \times C).$$

- (a) Let A be the interval $[1, 3]$, B be the interval $[2, 4]$, and C be the interval $[-1, 1]$. Draw the coordinate axes for \mathbb{R}^2 , and indicate the set $A \cap B$ on the x -axis. Outline and shade in the region $(A \cap B) \times C$ in \mathbb{R}^2 .
(b) On a separate graph, outline the regions $(A \times C)$ and $(B \times C)$ in \mathbb{R}^2 and shade in the region $(A \times C) \cap (B \times C)$. Use these graphs to make sure you believe that Theorem 2.45(b) is true.
(c) Prove Theorem 2.45(b).

- (5) (10 points)

- (a) Sketch the sets X given below (some are in \mathbb{R}^2 and others are in \mathbb{R}^3 . I know drawing in \mathbb{R}^3 is challenging. Do your best.) Here I am using S^1 to denote the unit circle.

$$S^1 = \{(x, y) \in \mathbb{R}^2 | \sqrt{x^2 + y^2} = 1\}$$

- (i) $X = \{(x, y) \in \mathbb{R}^2 | y = 0\} \times \{3\}$.
(ii) $X = S^1 \times \mathbb{R}$.
(iii) $X = \mathbb{R} \times S^1$.
(iv) Let $W = \{\frac{m}{5} | m \in \mathbb{Z}\}$. $X = \mathbb{R} \times W$.

- (6) (10 points) Explain how you can tell that the diagram shown below is not a Cartesian product of two sets in \mathbb{R} . Find sets A and B in \mathbb{R} such that $(A \times A) \cup (B \times B)$ is the set shown below.

