Newberger: Math 233, Spring 2004

Tips for finishing Worksheet: Primes.

(1) In this assignment, you will prove Corollary 6.27, which says: Let p be prime, and let $a_1 \dots a_n$ be integers. If $p|a_1 \dots a_n$, then $p|a_i$ for some i with $1 \le i \le n$.

Let $P(n)$ be the statement:

If p divides any product of n integers (i.e. if $p|a_1 \dots a_n$), then p divides one of the integers (i.e. then $p|a_i$ for some i with $1 \le i \le n$),

or more shortly, let $P(n)$ be the statement:

If $p|a_1 \dots a_n$, then $p|a_i$ for some $1 \le i \le n$.

- If you misunderstood $P(n)$, or stated $P(1)$ and $P(2)$ incorrectly, read this bullet carefully!

  The statement $P(n)$ is an "if-then" statement. It says "If $p|a_1 \dots a_n$, then something happens." It does not say that p actually divides $a_1 \dots a_n$. So for example, P(3) says:

  If $p|a_1a_2a_3$, then $p|a_i$ for some $1 \le i \le 3$.

  If we know that $P(3)$ is true, we do **not** know that $p|a_1a_2a_3$ is true. However

  If we know that $P(3)$ is true and that $p|a_1a_2a_3$,

  then we could conclude that the "then" part of $P(3)$ holds: i.e.

  then we could conclude that $p|a_i$ for some $1 \le i \le 3$.

  Upshot: Make sure to put the "if-then" in the right place when you state $P(1)$, $P(2)$, $P(k)$ and $P(k+1)$.

- Setting up the induction step.

  Induction Step Suppose that $P(k)$ holds. This means that

  If $p|a_1 \dots a_k$, then $p|a_i$ for some $1 \le i \le k$.

  This is an "if-then" statement. To use it, you must first show that the "if" part holds. Then you will be able to conclude that the "then" part holds.

  We want to show $P(k+1)$ holds. This means we want to show

  If $p|a_1 \dots a_{k+1}$, then $p|a_i$ for some $1 \le i \le k+1$.

  To prove this, we will assume the "if" part holds, and prove the "then" part holds. So we assume that $p|a_1 \dots a_{k+1}$, and we want to prove that $p|a_i$ for some $1 \le i \le k+1$.

  Upshot: So we assume that $p|a_1 \dots a_{k+1}$ and that $P(k)$ holds, and we want to prove that $p|a_i$ for some $1 \le i \le k+1$.

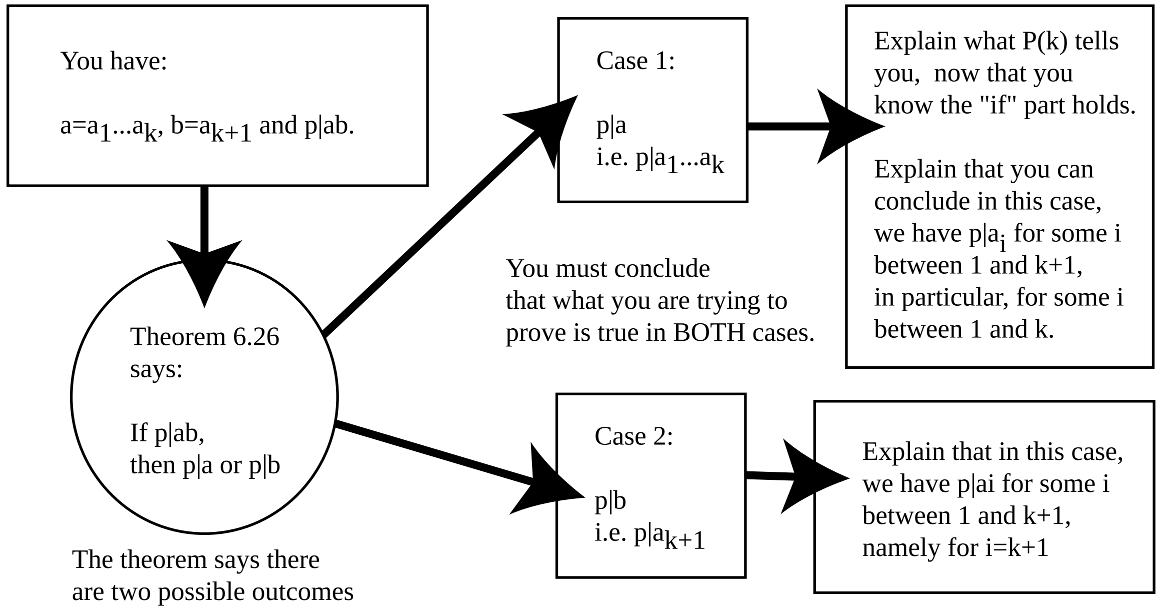
- Applying Theorem 6.26.

  Now in the worksheet, I suggested that you consider $a_1 \dots a_{k+1}$ as the product of two numbers. Let $a = a_1 \dots a_k$ and let $b = a_{k+1}$. Then we can change $p|a_1 \dots a_{k+1}$ into $p|ab$. This is helpful because we want to apply Theorem 6.26, which says:

  Let p be prime, and let $a, b \in \mathbb{Z}$. If $p|ab$, then $p|a$ or $p|b$.

  Note that this is an "if-then" statement. To use an "if-then" statement, we make "if" part true. The theorem says when we have the "if" part, then we can conclude that the "then" part holds.

Upshot: We let $a = a_1 \dots a_k$ and $b = a_{k+1}$. Then our assumption that $p|a_1 \dots a_{k+1}$ becomes $p|ab$. By Theorem 6.26, we can conclude $p|a$ or $p|b$.



Do not write anything that does not make sense to you. If you need help making sense of this stuff, ask!

(2) Problem 17 on page 305: prove that $\sqrt[3]{5}$ is irrational.

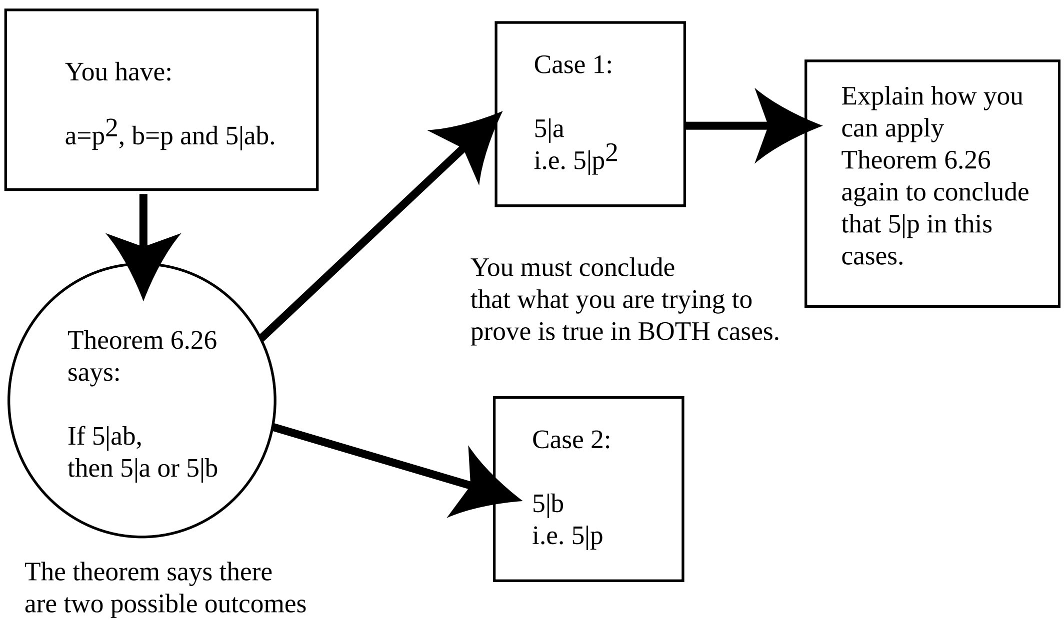
The proof of this statement is similar to the proof that $\sqrt{2}$ is irrational. We proceed by contradiction. Suppose that $\sqrt[3]{5} = \frac{p}{q}$, and suppose that the fraction is reduced. However instead of proving p and q are even as we did in the argument that $\sqrt{2}$, you will prove that $5|p$ and $5|q$. This will contradict the assumption that the fraction was reduced. We begin with

$$\sqrt[3]{5} = \frac{p}{q}$$

and rearranging, we have

$$p^3 = 5q^3$$

from which you can conclude that $5|p^3$. What we want to say is that $5|p$. You should explain how you can use Theorem 6.26 to get from $5|p^3$ to $5|p$.



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