

## Worksheet: Primes.

(1) (10 points) In this assignment, you will prove Corollary 6.27, which says:

*Let  $p$  be prime, and let  $a_1, \dots, a_n$  be integers. If  $p|a_1 \dots a_n$ , then  $p|a_i$  for some  $i$  with  $1 \leq i \leq n$ .*

This proof is by induction on  $n$ .

- Let  $p$  be a fixed prime number.
- In your argument, let  $P(n)$  be the statement:  
*If  $p$  divides any product of  $n$  integers (i.e. if  $p|a_1 \dots a_n$ ) then  $p$  divides one of the integers (i.e. then  $p|a_i$  for some  $i$  with  $1 \leq i \leq n$ .)*
- Write what  $P(1)$  says, and explain that it is true.
- Write what  $P(2)$  says, and explain how you know it is true (apply Theorem 6.26). This is the basis step for this problem.
- Now work on the induction step. Let  $k$  be a fixed integer with  $k \geq 2$ . State in your argument that you are assuming that  $P(k)$  holds and what it means.
- You want to prove  $P(k+1)$ . This means that you want to prove the following if-then statement:

*If  $p|a_1 \dots a_{k+1}$ , then  $p|a_i$  for some  $i$  with  $1 \leq i \leq k+1$ .*

– Write your assumptions:

Assume that  $p|a_1 \dots a_{k+1}$ , and that  $P(k)$  holds.

– Write what you want to prove:

We want to show  $p|a_i$  for some  $i$  with  $1 \leq i \leq k+1$ .

– Here is a hint about how to complete the proof:

Consider the product  $a_1 \dots a_{k+1}$  to be the product of  $a_1 \dots a_k$  with  $a_{k+1}$ . Apply Theorem 6.26, with  $a = a_1 \dots a_k$  and  $b = a_{k+1}$ . Explain what the theorem tells you about this situation. Use the induction hypothesis to complete the proof.

(2) (10 points) Another application of Theorem 6.26.

- Study the proof that  $\sqrt{2}$  is irrational given in class or found in the book (Theorem 6.31).
- Do problem 17 on page 305. Make sure that your argument is easy to follow and without gaps.
- Conjecture a generalization of the statement in problem 17 on page 305.