Newberger: Math 233, Spring 2004

Worksheet: Divides

(1) (10 points) Do page 303, #1. This is a generalization of Theorem 6.15(b). After you do the exercise, write a sentence saying what values of m and n would yield each of the statements in Theorem 6.15(b).

(2) (10 points) Do page 303 #2. If the statement is true, give a proof. If the statement is false, give a counterexample. To give a counterexample, name particular numbers satisfying the assumptions in the statement, but not the conclusion of the statement. (If the statement is $P \Rightarrow Q$, the assumption is P and the conclusion is Q.)

(3) (10 points) Let $a, b, c \in \mathbb{Z}$. Prove if $3a + 7c = 1$ and $a|bc$, then $a|b$.

(4) (10 points)

- Give an example to show that in order to prove $a|b$, it is not sufficient to assume that $a|bc$; i.e. give an example to show that the following statement is false:

  If $a|bc$, then $a|b$.
- Conjecture (i.e. state) a generalization of the statement you proved in (3), by modifying the assumption $3a + 7c = 1$.
- Prove your conjecture.

(5) (10 points) This exercise will show you what the assumption $3a + 7c = 1$ tells you about a and c. Let a and c be integers. Prove that if $3a + 7c = 1$, then the only integers that divide both a and c are 1 and -1.

Tips for setting it up: Suppose b is an integer that divides both a and c. You want to prove that b is either 1 or -1. In other words, prove the statement:

If a, b and c are integers such that $3a + 7c = 1$, $b|a$ and $b|c$, then $b = 1$ or $b = -1$.