Newberger: Math 233, Spring 2004

Worksheet: Contrapositive

Please refer to Section 1.4 of *Structures and Proofs*. You do not need to copy out the questions, but answer the following exercises in sentences, so that it is not necessary to read the questions to understand what you have written.

1. (10 points) In this exercise, you will show that the indirect proof method called contrapositive is valid. In particular you will show that $P \Rightarrow Q$ and $\sim Q \Rightarrow \sim P$ are logically equivalent.

- Reread the end of Section 1.4 beginning with the paragraph beginning, "A special...," at the bottom of page 25 of *Structures and Proofs*.
- Look at the truth table numbered 1.18 at the bottom of page 14 of *Structures and Proofs*, and read the paragraph preceding truth table 1.18 through to the end of the section. Write a sentence or two explaining when $P \Leftrightarrow Q$ is true. (By "when" we mean for which truth-values of P and Q.)
- Suppose P and Q are sentential variables, and let S_1 be the sentential form P implies Q ($P \Rightarrow Q$) and S_2 be the sentential form not Q implies not P ($\sim Q \Rightarrow \sim P$). Write truth tables for S_1 and S_2. (Please label your tables).
- Now combine the information in the tables for S_1 and S_2 to construct a truth table whose columns are P, Q, S_1, S_2 and $S_1 \Leftrightarrow S_2$.
- Read Section 1.5, in particular Definitions 1.24 and 1.26. Example 1.27 should also be helpful.
- Look at the table for $S_1 \Leftrightarrow S_2$ that you constructed. Are $P \Rightarrow Q$ and $\sim Q \Rightarrow \sim P$ logically equivalent? Justify your answer by referring to what you see in the table.
- Write a sentence to conclude in general a) what truth table you would construct to determine whether or not sentential forms $S1$ and $S2$ are logically equivalent, and b) what features the table has when the two statements are logically equivalent.

2. (10 points) In this exercise you will prove the following statement about an integer n:

$$n \textit{ is odd if and only if } n^2 + 1 \textit{ is even.}$$

Let P be the statement "n is odd," and let Q be the statement "$n^2 + 1$ is even." Thus we will show $P \Leftrightarrow Q$ ("if and only if" and "$\Leftrightarrow$" have the same meaning). To show $P \Leftrightarrow Q$, you must complete two proofs: $P \Rightarrow Q$ and $Q \Rightarrow P$.

Though you do not need to copy what is written above word for word onto your homework, you should write something that includes the statement we are interested in and the definitions of P and Q. There are some guidelines on the next page that may help you organize your thoughts.

- Prove $P \Rightarrow Q$ directly. Directly means that you should assume that P is true and prove that Q is true.
- State the contrapositive of $Q \Rightarrow P$ in words.
- Prove $Q \Rightarrow P$ by proving the contrapositive statement. This means you should assume that $\sim P$ is true and prove $\sim Q$ is true.

Guidelines to help you discover and explain a proof (please do not enumerate these steps in your proofs):

(1) Write down your assumptions. Explain what they mean in as much detail as you can. Refer directly to the definitions of the vocabulary in the statement.
(2) Write down what you want to prove. Again refer to the definitions of the words in the statement. Be sure to write the words"We want to prove..." to clearly distinguish what you want to prove from what you are assuming.
(3) Use (1) to show (2).