Newberger: Math 233, Spring 2004

Worksheet: Relations.

Instructions for numbers (1) through (5): Determine whether or not each of the following relations is reflexive, symmetric and/or transitive. Prove your answers are correct. If the given relation is an equivalence relation, describe the equivalence classes.

(1) (10 points) Let R be the relation on $\mathbb{R}$ given by $R = \{(x, y)|x^2 = y^2\}$.

(2) (10 points) Let R be the relation on $\mathbb{R}$ given by $R = \{(x, y)|0 < xy\}$.

(3) (10 points) Let $n \in \mathbb{N}$. Let R be the relation on $\mathbb{Z}$ given by $R = \{(x, y)|\ n|(x - y)\}$.

(4) (10 points) Let R be the relation on $\mathbb{Z}$ given by $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z}|\ a|b\}$.

(5) (10 points) For this part, you need to be able to calculate the distance between two points in the complex plane. Recall that if $z = a + bi$, then $|z| = \sqrt{a^2 + b^2}$. Let $z, w \in \mathbb{C}$. Then the distance between z and w is given by

$$dist(z, w) = |z - w|.$$

Warm-up:

(a) What is the distance between $2i$ and $4i + 2$?

(b) Draw a diagram showing the set $\{z \in \mathbb{C}|dist(z, 2i) < 1\}$.

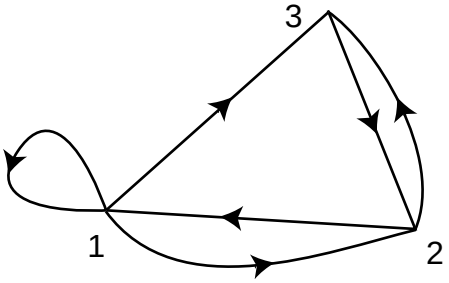
Let R be the relation on $\mathbb{C}$ given by

$$R = \{(z_1, z_2)|dist(z_1, z_2) < 1\}.$$

(6) (10 points) Recall that $\mathbb{N}_m = \{1, 2, ..., m\}$. (So $\mathbb{N}_3 = \{1, 2, 3\}$, for example).

(a) Referring to the graph below, let R be the relation on $\mathbb{N}_3$ given by

$$R = \{(x, y) \in \mathbb{N}_3 \times \mathbb{N}_3|\text{there is an edge pointing from } x \text{ to } y\}.$$



(i) List the elements of R.

(ii) Determine whether or not R is reflexive, symmetric and/or transitive, and prove your answer is correct.

(b) For each item below, draw a graph with 5 vertices so that the relation

$$R = \{(x, y) \in \mathbb{N}_5 \times \mathbb{N}_5|\text{there is an edge pointing from } x \text{ to } y\}.$$

on $\mathbb{N}_5$ has the given properties.

(i) Reflexive, symmetric and transitive.

(ii) Not reflexive, symmetric, and not transitive.

(iii) Reflexive, not symmetric, and transitive.