Section 3.1 #1 *Prove that if $a|b$ and $a|c$ then $a|(mb + nc)$ for all $m, n \in \mathbb{Z}$.*

Begin by writing what your assumptions are. Expand your assumptions by appealing directly to the definitions involved.

> *We know that $a|b$ and $a|c$. This means that there exists integers k and l such that $b = ak$ and $c = al$.*

Now explain what you want to prove, again referring to the definitions. Be careful not use the statements that you want to prove in your argument.

> *We want to show $a|(mb + nc)$. This means we want to find an integer d such that $mb + nc = ad$.*

Again, do not use the equation $mb + nc = ad$ in your calculations. Instead, start simply with the left hand side of this equation. We are interested in $mb + nc$, so begin with that. Write

$$mb + nc = ...$$

Plug in $b = ak$ and $c = al$, and factor out an a. Say what d should be in order to have $mb + nc = ad$. Conclude that $a|(mb + nc)$.

You must also write a sentence saying what values of m and n would yield each of the statements in Theorem 6.15(b). Here is an example to help you figure out what this means.

Example: What values of m and n would yield each of the following two statements: if $a|b$ and $a|c$, then $a|(3b + 2c)$ and $a|(2b - 3c)$?

Answer: The statement we proved becomes if $a|b$ and $a|c$, then $a|(3b + 2c)$ when $m = 3$ and $n = 2$. The statement we proved becomes if $a|b$ and $a|c$, then $a|(2b - 3c)$ when $m = 2$ and $n = -3$.