Newberger Math 233 Spring 2004

MIDTERM EXAM REVIEW

This exam will cover Chapter 2, Sections 4,5, and 8, and Chapter 3, Sections 1, 2, 3. In addition, this exam will cover limits.

(1) The following list consists of terms that you may be asked to define.
- Subset (page 57).
- Union, intersection and difference of sets (page 62-3).
- Cartesian product (page 83).
- Domain, image, pre-image (page 118).
- Identity mapping and constant mapping (page 119).
- One-to-one, onto, injection, surjection, bijection (page 127-8).
- Composition of functions (page 141).
- Commutative/commute (page 143).
- Graph of a function.

  The graph of a function $f : A \to B$ is the set $\{(a, b) \in A \times B | b = f(a)\}$.
- Inverse function (page 145). The definition in the text is not stated so clearly. Here is a clear definition of an inverse function.

  Let A, B be sets and let $f : A \to B$. The inverse of f is a function $f^{-1} : B \to A$ such that $f \circ f^{-1} = i_B$ and $f^{-1} \circ f = i_A$.

  Here i_A is the identity map from A to A, and i_B denotes the identity function from B to B.
- The limit of a function as x approaches a finite number c.

  Let (a, b) be an interval in $\mathbb{R}$. Let $f : (a, b) \to \mathbb{R}$. Let $c \in (a, b)$. We say L is the limit of $f(x)$ as x approaches c if for every $\varepsilon > 0$, there exists a number $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$. We write

  $$L = \lim_{x \to c} f(x).$$
- The limit of a function as x approaches $+\infty$.

  Let $(a, +\infty)$ be an interval in $\mathbb{R}$. Let $f : (a, +\infty) \to \mathbb{R}$. We say L is the limit of $f(x)$ as x approaches $+\infty$ if for every $\varepsilon > 0$, there exists a number K such that if $x > K$, then $|f(x) - L| < \varepsilon$. We write

  $$L = \lim_{x \to +\infty} f(x).$$

(2) Tip: Study your homework assignments, and the activities completed in class. The exams will focus on the same material as the homework assignments do.

(3) **Guarantees.** Your exam will have from 4 to 6 problems, some consisting of several parts.
- (30%) You will be asked to state some of the definitions in (1). Definitely know the definition of the limits.
- (70%) You will be asked to prove statements, answer questions and provide examples involving the material on the lists above. The proofs will be in the style of the homework problems, involving the material listed in (1) above. In particular:
  - You **will** be asked to prove that the limit of a given function is equal to a given number.
  - You **will** be asked questions about elements of sets, possibly including unions, intersections, cartesian products. Review closed under addition and multiplication. Be able to sketch sets of numbers on the number line (in $\mathbb{R}$), in $\mathbb{R}^2$ or $\mathbb{R}^3$.
  - You **will** be asked to show that two sets are equal or that one set is contained in another set.
  - You may be asked to prove that a "for every" statement is false; for example that a set is not closed under multiplication, or two functions are not equal.
  - You **will** be asked to show a function is or is not one-to-one and/or onto. You may be asked to calculate the image or inverse image of a set under a function.

(4) On a problem in which you are asked to write a proof, you will receive substantial partial credit for correctly setting up the problem and for completely explaining what you want to prove and what your assumptions are, even if you are not able to complete the problem.