

Exam 1 Review Sheet

This exam covers material from Gerstein Sections 1.1-1.5, 2.1-2.3, 2.10, and 6.3.

- (1) You should be familiar with all of the following theorems and vocabulary. You **will be asked** to give precise mathematical statements for some of the vocabulary listed below.

- Even and odd integer. This is not stated explicitly in the text, so I state the definitions here.

An integer m is odd if $m = 2k + 1$, for some integer k .

An integer n is even if $n = 2l$, for some integer l .

- Closed under addition or multiplication. This is not in the text, so I state the definitions here.

A set S is closed under addition if the following statement is true:

$$x \in S \text{ and } y \in S \Rightarrow x + y \in S.$$

A set S is closed under multiplication if the following statement is true:

$$x \in S \text{ and } y \in S \Rightarrow xy \in S.$$

- De Morgan's Laws (Equation 1.28 page 29).
- Principle of mathematical induction (Third Version) (page 109).
- Divides (page 291).
- Division Algorithm (page 293).
- Greatest common divisor (page 294).
- Prime (page 4).

- (2) From Chapter 1:

- You may be asked to use English sentences to represent sentential forms, without using phrases like "it is not true that" or "it does not hold that." Be able to apply De Morgan's Laws (Equation 1.28 page 29) to get rid of parentheses.
- Given a statement in the form $P \Rightarrow Q$, you may be asked to state the contrapositive.
- You may be asked to explain what assumptions you make and what you are trying to prove when proving $P \Rightarrow Q$ using each of the following techniques: i) direct proof, ii) proof by contradiction, and iii) proving the contrapositive.
- You may be asked to prove a statement of the form P if and only if Q ($P \Leftrightarrow Q$). You should know that this means you must do two proofs: $P \Rightarrow Q$ and $Q \Rightarrow P$.
- You may be asked to state the negation of statements of the form $P \Rightarrow Q$, $P \wedge Q$ and/or $P \vee Q$, without using the words "it is not true that."

(3) From Chapter 2:

- Given a set S , you may be asked to determine whether or not a given object is an element of S . You may be asked to give examples of elements of S .
- You may be asked to determine the validity of mathematical statements involving sets (True or False questions). For questions such as page 47 #5, you will not be asked to justify your answers.
- You will be asked to prove a given set is closed under multiplication and/or addition.
- You may be asked to negate a statement involving for every (\forall) or there exists (\exists).
- You will be asked to prove a statement by induction (using version 3 page 109 for an appropriate choice of N).

(4) From Section 6.3:

- You may be asked to determine the validity of mathematical statements (True or False questions). For a false statement you will be asked to state a counterexample to show a statement is false. For a true statement, you will be asked to give a proof, or to state a theorem which implies the statement holds.
- You may be asked to generalize a given statement.
- You will be asked to prove statements involving divisibility.
- You may be asked to prove a statement involving the greatest common divisor of two numbers. Make sure you know Corollary 6.21.
- You may be asked to prove a statement about prime numbers. Make sure you know Theorem 6.26.

(5) Guarantees: Students will not be permitted to bring any materials into the exam, with the exception of pencils and erasers. University standards for academic integrity will be enforced.

- (a) 30% of the exam will be stating definitions and theorems chosen from those in bold in (1) above. You are not required to use the exact words given in the book or in lecture, but your definitions and statements must be correct, precise mathematical statements. I suggest memorizing them.
- (b) The test will probably have 5 pages. Each page will typically consist of one problem in several parts. For example, I may ask you (a) state a definition, (b) prove something involving that definition, and (c) answer a related question, or (a) state the contrapositive of a given statement and then (b) prove the statement.