Warm-up

We have a bag containing 9 red beans and 1 green bean and nothing else.

True or False:

1. The bag contains 7 red jelly beans.
   True.
2. Every bean in the bag is red.
   False. There exist beans in the bag that are not red.
3. None of the beans in the bag are red.
   False. There exist beans in the bag that are red.

More Warm-up

True or False:

1. If I were the president, then I would live in the white house.
   True.
2. If I were not the president, then I would not live in the white house.
   False. There are people other than the president that live in the white house.
3. If I do not live in the white house, then I am not the president.
   True.
4. If I live in the white house, then I am the president.
   False. Same reason as (2).

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

True or False:

1. $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^m$.
   False. There are $\mathbf{b}$*'s that would make it inconsistent, for example, it would be inconsistent if* $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.
2. $A\mathbf{x} = \mathbf{b}$ is inconsistent for every $\mathbf{b}$ in $\mathbb{R}^m$.
   False. There are $\mathbf{b}$*'s that would make it consistent, for example, it would be consistent if* $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

True or False:

1. For an $m \times n$ matrix A, $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^m$.
   False. If A *has a pivot in every row, then* $A\mathbf{x} = \mathbf{b}$ *is consistent for every* $\mathbf{b}$ *in* $\mathbb{R}^m$.
2. If $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^m$, then A has a pivot position in every row.
   True. The statement,

   "$A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^m$"

   is equivalent to the statement,

   "A has a pivot position in every row."

   These statements are either both true or both false.

3. If A does not have a pivot position in every row, then $A\mathbf{x} = \mathbf{b}$ is inconsistent for every $\mathbf{b}$ in $\mathbb{R}^m$.

   False. Even if A does not have a pivot position in every row, there will still be a vector $\mathbf{b} \in \mathbb{R}^m$ such that the matrix equation $A\mathbf{x} = \mathbf{b}$ is consistent. Note that for any matrix A, the equation $A\mathbf{x} = \mathbf{0}$ is consistent.

   In this problem, this statement is False:

   "A has a pivot position in every row."

   So this statement is also False:

   "$A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^m$"

   However, that does not imply that this statement is True:

   "$A\mathbf{x} = \mathbf{b}$ is inconsistent for every $\mathbf{b}$ in $\mathbb{R}^m$"

4. If A does not have a pivot position in every row, then there exists a vector $\mathbf{b}$ in $\mathbb{R}^m$ such that $A\mathbf{x} = \mathbf{b}$ is inconsistent.

   True.

   In this problem, this statement is False:

   "A has a pivot position in every row."

   So this statement is also False:

   "$A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^m$"

   This implies that this statement is True:

   "There exists a vector $\mathbf{b}$ in $\mathbb{R}^m$ such that $A\mathbf{x} = \mathbf{b}$ is inconsistent."