

Quiz 4.1-4.2

name: _____

1. (12 points) Show that the following set is not a subspace of \mathbb{R}^3 .

$$H = \left\{ \left[\begin{array}{c} a + b - 3 \\ a - b \\ 2b + 1 \end{array} \right] \mid a, b \in \mathbb{R} \right\}$$

We will show that the zero vector $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is not in H . To see if $\mathbf{0}$ is in H , we will try to find a and b such that

$$\begin{bmatrix} a + b - 3 \\ a - b \\ 2b + 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

To solve this system using linear algebra, we must move all the constants to the right hand side of the equation. We get

$$\begin{bmatrix} a + b \\ a - b \\ 2b \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}.$$

This becomes

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}.$$

Reducing, we get

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & -1 & 0 \\ 0 & 2 & -1 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -2 & -3 \\ 0 & 2 & -1 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -2 & -3 \\ 0 & 0 & -4 \end{array} \right]$$

which shows that the system is inconsistent. Thus there are no real numbers a and b such that

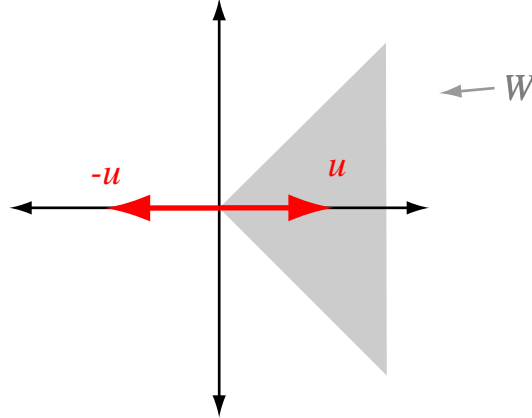
$$\begin{bmatrix} a + b - 3 \\ a - b \\ 2b + 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

This means $\mathbf{0}$ is not in H and H is not a subspace.

2. (12 points) Show that the following set is not a subspace of \mathbb{R}^2 .

$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid -x \leq y \leq x \right\}$$

Here is a graph of the region W .



Note that the vector $u = (1, 0)$ is in W , since $-1 \leq 0 \leq 1$. However $-u = (-1, 0)$ is not in W , since $1 \leq 0 \leq -1$ is false. Thus W is not closed under scalar multiplication.