

**Quiz 4.1-4.2**

name: \_\_\_\_\_

1. State the definition of a subspace of  $\mathbb{R}^n$ .

*A subspace of  $\mathbb{R}^n$  is any set  $H$  in  $\mathbb{R}^n$  that has three properties:*

*(a) The zero vector is in  $H$ .*

*(b) For each  $\mathbf{u}$  and  $\mathbf{v}$  in  $H$ , the sum  $\mathbf{u} + \mathbf{v}$  is in  $H$ .*

*(c) For each  $\mathbf{u}$  in  $H$  and each scalar  $c$ , the vector  $c\mathbf{u}$  is in  $H$ .*

2. Show that the following set is a subspace of  $\mathbb{R}^3$ .

$$H = \left\{ \begin{bmatrix} a + 2b + c - 3d \\ 2a + b - c \\ b + c - 2d \end{bmatrix} \mid a, b, c, d \in \mathbb{R} \right\}$$

*$H$  is the set of all vectors that can be written in the form:*

$$\begin{bmatrix} a + 2b + c - 3d \\ 2a + b - c \\ b + c - 2d \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} a + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} b + \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} c + \begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix} d.$$

*Thus  $H$  is the set of all linear combinations of the vectors*

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix}.$$

*So*

$$H = \text{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix} \right\},$$

*which means  $H$  is a subspace, since all spans are subspaces.*

3. Show that the following set is a subspace of  $\mathbb{R}^3$ .

$$W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \mid \begin{array}{l} a + 3b - c = 0 \\ b + c + a = 0 \end{array} \right\}$$

*Elements in  $W$  are vectors  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  that satisfy*

$$a + 3b - c = 0$$

$$b + c + a = 0,$$

*which can be rewritten as the following matrix equation:*

$$\begin{bmatrix} 1 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Thus  $W$  is the set of all solutions to the homogeneous equation associated to the matrix  $A = \begin{bmatrix} 1 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ , i.e.  $W = \text{Nul } A$ . So  $W$  is a subspace since null spaces are subspaces.