Newberger Math 247 Spring 03

Quiz 2.1-2.3 name:____________________

1. Find the inverse of $A = \begin{pmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 2 & 2 \end{pmatrix}$.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -5 & 1 & 0 \\ 0 & 2 & 2 & 0 & 0 & 1 \end{array}\right)$$
$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -5 & 1 & 0 \\ 0 & 0 & 2 & -10 & -2 & 1 \end{array}\right)$$

Thus

$$A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ -5 & -1 & \frac{1}{2} \end{pmatrix}.$$

2. Suppose A is invertible and $AB = \mathbf{0}$, where $\mathbf{0}$ denotes the zero matrix. Show that $B = \mathbf{0}$.

Since A is invertible, we have the matrix A^{-1} to work with. Multiplying $AB = \mathbf{0}$ by A^{-1}, we get $A^{-1}(AB) = A^{-1}\mathbf{0}$. Using the associative law, this becomes $(A^{-1}A)B = A^{-1}\mathbf{0}$. But $A^{-1}A = I$ and the right hand side is $\mathbf{0}$, so we get $B = \mathbf{0}$.

3. Suppose $B = PAP^{-1}$, where A and P are invertible $n \times n$ matrices. Find B^{-1}.

The inverse of the product of invertible matrices is the product of the inverses in reverse order.

$$\begin{aligned} B^{-1} &= (PAP^{-1})^{-1} \\ &= (P^{-1})^{-1}A^{-1}P^{-1} \\ &= PA^{-1}P^{-1} \end{aligned}$$

4. Let A and B be invertible $n \times n$ matrices. Explain why the columns of $A^{-1}B$ span all of $\mathbb{R}^n$.

Since A is invertible, so is A^{-1} (its inverse is A). Now since A^{-1} and B are invertible, so is $A^{-1}B$ (since the product of invertible matrices is invertible). Now by the invertible matrix theorem, $A^{-1}B$ has n pivot positions so it has a pivot in every row, and the columns of $A^{-1}B$ span all of $\mathbb{R}^n$.