Newberger Math 247 Spring 03

Quiz 5 over Section 1.9 name:________________________

1. Let T be the linear transformation that rotates clockwise by $\pi/2$ radians and then reflects across the x_1 axis. Find the standard matrix for T.

   Find the standard matrix by calculating $T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, *and* $T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. *Then the standard matrix is* $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. *This has a pivot in every row and every column, so it is both one-to-one and onto.*

2. Let $A = \begin{bmatrix} 3 & 3 \\ 3 & 4 \\ 2 & 2 \end{bmatrix}$ and let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be given by $T(\mathbf{x}) = A\mathbf{x}$. Is T one-to-one? Is T onto? How do you know?

   A reduces to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$. *It is one-to-one since it has a pivot in every column, and it is not onto since there is a row without a pivot.*

3. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is linear and onto. Explain why T must also be one-to-one.

   Since T *is linear, there is a* 3×3 *matrix* A *such that* $T(\mathbf{x}) = A\mathbf{x}$. *Since* T *is onto,* A *has a pivot in every row. But since* A *is* 3×3 A *also has a pivot in every column. Thus* T *is one-to-one.*

4. Give an example of *transformation* that is onto but not one-to-one.

   Let m *and* n *be positive integers with* $n > m$. *Let* $T : \mathbb{R}^n \to \mathbb{R}^m$ *be given by* $T(\mathbf{x}) = A\mathbf{x}$ *where* A *is any* $m \times n$ *matrix having a pivot position in every row. Then* T *is onto but not one-to-one. For example, we could take* $T : \mathbb{R}^3 \to \mathbb{R}^2$ *be given by* $T(\mathbf{x}) = A\mathbf{x}$ *where* $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.